24th Summer Conference on Topology and Its Applications

July 14 – 17, 2009

Brno, Czech Republic

Hosted by

Faculty of Electrical Engineering and Communication

Brno University of Technology

Program | Book of abstracts

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The **24th Summer Conference on Topology and Its Applications** is organized with the support of the City of Brno, the South Moravian Region and the Brno University of Technology. The Conference is hosted by the Faculty of Electrical Engineering and Communication, Brno University of Technology on the occasion of the 50-th anniversary of the foundation of the Faculty and the 110-th anniversary of the foundation of the University.

B R N O

Jihomoravský kraj

BRNO UNIVERSITY OF TECHNOLOGY

FACULTY OF ELECTRICAL ENGINEERING AND COMMUNICATION

24th Summer Conference on Topology and Its Applications

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Program

General Topology 1	Set-Theoretic Topology	Topology in Computer Science
General Topology 2	Categorical Topology	Topology in Algebra

Tuesday, July 14

Time Room	9.00 - 9.30	9.30 - 10.00	10.00 - 10.30	10.30 - 11.00	11.00 - 11.30	11.30 - 12.00	12.00 - 12.30	12.30 - 13.00	13.00 - 13.30
P381	Opening	A. Dow		Coffee break	E. Ivanova	K. Belaid	M.R.A. Zand	Lunch	Lunch
E337				Coffee break	A. Szymanski	M. Demirci	G. Dimov	Lunch	Lunch
E339				Coffee break	G. Lukacs	J. Galindo	L. Aussen-hofer	Lunch	Lunch
E340				Coffee break				Lunch	Lunch
E342				Coffee break	M. Kovár	T. Richmond	I. Pratt-Hartmann	Lunch	Lunch
E109				Coffee break	R. Beattie	M.R. Buneci		Lunch	Lunch
E110				Coffee break	M. Črepnjak	I. Banič	Z. Kočan	Lunch	Lunch

Time Room	13.30 - 14.00	14.00 - 14.30	14.30 - 15.00	15.00 - 15.30	15.30 - 16.00	16.00 - 16.30	16.30 - 17.00	17.00 - 17.30	17.30 - 18.00
P381	Lunch	J.T. Moore		S. Lazaar	S. Hussain	Coffee break	D. Georgiou	G. Bezhanishvili	
E337	Lunch				A. Megaritis	Coffee break	J. Chvalina		
E339	Lunch			R. Frič	J. Paseka	Coffee break	S. Andima		
E340	Lunch					Coffee break			
E342	Lunch			S. Matthews	M. Bukatin	Coffee break			
E109	Lunch			J. Kakol	H. Junnila	Coffee break			
E110	Lunch			C. Good	A. Barwell	Coffee break	R. Hric		

Topology in Geometry	Topology in Dyn. Systems
Topology in Funct. Analysis	Plenary Lecture

Wednesday, July 15

Time Room	9.00 - 9.30	9.30 - 10.00	10.00 - 10.30	10.30 - 11.00	11.00 - 11.30	11.30 - 12.00	12.00 - 12.30	12.30 - 13.00	13.00 - 13.30
P381	B. Cascales		Coffee break	L.R. Rubin	M. Patrakeev	M. Sakai	Lunch	Lunch	Lunch
E337			Coffee break	V. Gregori	S. Morillas	H.M. Golshan	Lunch	Lunch	Lunch
E339			Coffee break	Dibyendu De	T.C. Stevens		Lunch	Lunch	Lunch
E340			Coffee break	M.M. Clementino	W. Rosiers	G. Gutierres	Lunch	Lunch	Lunch
E342			Coffee break	I. Juhász	L. Zdomskyy	V. Tkachuk	Lunch	Lunch	Lunch
E109			Coffee break	M. Joita	O. Kalenda	S.A. Saxon	Lunch	Lunch	Lunch
E110			Coffee break	J.C. Mayer	R. Ptacek	M.C. Sullivan	Lunch	Lunch	Lunch

Time Room	13.30 - 14.00	14.00 - 14.30	14.30 - 15.00	15.00 - 15.30	15.30 - 16.00	16.00 - 16.30	16.30 - 17.00	17.00 - 17.30	17.30 - 18.00
P381									
E337									
E339									
E340					**Excursion**				
E342									
E109									
E110									

General Topology 1	Set-Theoretic Topology	Topology in Computer Science
General Topology 2	Categorical Topology	Topology in Algebra

Thursday, July 16

Time Room	9.00 - 9.30	9.30 - 10.00	10.00 - 10.30	10.30 - 11.00	11.00 - 11.30	11.30 - 12.00	12.00 - 12.30	12.30 - 13.00	13.00 - 13.30
P381	W. Tholen		Coffee break	O. Okunev	D. Basile	A. Osipov	O. Echi	Lunch	Lunch
E337			Coffee break	J. Slovák	J. Hrdina	L. Zalabová	M. Kureš	Lunch	Lunch
E339			Coffee break	J. Rodriguez -Lopez	A. Frolova	E. Bouassida	S. Han	Lunch	Lunch
E340			Coffee break	R. Börger	A. Van Geenhoven	T. Vroegrijk		Lunch	Lunch
E342			Coffee break	E. Tachtsis	Z. Mu-shaandja	D. Leseberg	M. Kada	Lunch	Lunch
E109			Coffee break	M. Lopez Pellicer	V. Montesinos	W. Kubis	S. Moll	Lunch	Lunch
E110			Coffee break	J. Kennedy	K. Kuperberg	C .Mouron	M. Tuncali	Lunch	Lunch

Time Room	13.30 - 14.00	14.00 - 14.30	14.30 - 15.00	15.00 - 15.30	15.30 - 16.00	16.00 - 16.30	16.30 - 17.00	17.00 - 17.30
P381	Lunch	J. Rosický		L. Mdzina-rishvili	A. Beridze	Coffee break	S. Dolecki	
E337	Lunch			K. Bauman	O. Spáčil	Coffee break	J. Kedra	
E339	Lunch			R. Kopperman	I. Rewitzky	Coffee break		
E340	Lunch			S. Paoli	V. Baladze	Coffee break		
E342	Lunch			A. Gryzlov	E. Bastrykov	Coffee break		
E109	Lunch			T. Rodionov	M. Wojtowicz	Coffee break		
E110	Lunch			H. Bruin	S. Štimac	Coffee break		

Topology in Geometry	Topology in Dyn. Systems
Topology in Funct. Analysis	Plenary Lecture

Friday, July 17

Time Room	9.00 - 9.30	9.30 - 10.00	10.00 - 10.30	10.30 - 11.00	11.00 - 11.30	11.30 - 12.00	12.00 - 12.30	12.30 - 13.00
P381	D. Hofmann		Coffee break	G. Tironi	P. Holický	R.W. Heath	M. Diker	Lunch
E337			Coffee break	V. Matijevic	T. Atanasova-Pacemska	E.D. Yildirim	S. Nada	Lunch
E339			Coffee break		H.-P. Künzi	C.M. Kivuvu	J. Šlapal	Lunch
E340			Coffee break	J. Petit	M.J. Ferreira	T. Kubiak		Lunch
E342			Coffee break	J. Brendle	L. Soukup	D. Chodounsky	M. Hrusak	Lunch
E109			Coffee break	S. Hernández	A.R. Todd	A. Leiderman	H. Mazaheri	Lunch
E110			Coffee break	A. Illanes	P. Krupski	J. Nikiel	V. Martínez de la Vega	Lunch

Time Room	13.00 - 13.30	13.30 - 14.00	14.00 - 14.30	14.30 - 15.00	15.00 - 15.30	15.30 - 16.00	16.00 - 16.30	16.00 - 16.30
P381	Lunch	Lunch		S.D. Iliadis	D. Shakhmatov		Coffee break	
E337	Lunch	Lunch	M. Basher	G. Günel			Coffee break	
E339	Lunch	Lunch					Coffee break	
E340	Lunch	Lunch					Coffee break	
E342	Lunch	Lunch	K.P. Hart	D. Gauld			Coffee break	
E109	Lunch	Lunch	M. Morillon				Coffee break	
E110	Lunch	Lunch	J. Bobok	P. Oprocha			Coffee break	Problem Session

Abstracts

Contents

General Topology

Maps of Quasicomponents Induced by a Shape Morphism

Tatjana Atanasova – Pacemska
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Abstract: The spaces considered are metric locally compact and separable. By $Q(X)$ we denote the space of quasicomponents. Using the intrinsic definition of shape, we give a positive answer to a question stated at the Borsuk Conference, 2005, Poland, by Nikita Schekutkovski: is it possible for noncompact spaces to prove the analog of the Borsuk's theorem for components of a compact metric space i.e. is ot true that for a shape morphism f from X to Y, there exists an unique map $f^\wedge : Q(X) \longrightarrow Q(Y)$ such that the restriction of f from Q to $f^\wedge(Q)$ is also a shape morphism?

This is a joint work with prof. Nikita Shekutkovski, Faculty of Natural Sciences and Mathematics, "St. Ciril and Methodius" University-Skopje, Macedonia.

σ-coloring of the Monohedral tiling

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In this paper we introduce the definition of σ-coloring and perfect σ-coloring for the plane which is equipped by tiling η. And we investigate the σ-coloring for the r-monohedral tiling.

On some open questions related to the κ-Ohio completeness property

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Coauthors: Jan van Mill

In 2005, Arkhangel'skii introduced the Ohio completeness property. We generalize it by introducing the κ-Ohio completeness property. For a fixed infinite cardinal κ, we say that a space X is κ-Ohio complete if for every compactification γX of X there is a G_κ-subset S of γX such that $X \subseteq S$ and for every $y \in S \setminus X$, there is a G_κ-subset of γX that contains y and misses X. So far, the following are the main open questions:

Question 1. Is κ-Ohio completeness closed-hereditary?

Question 2. Is κ-Ohio completeness finitely productive?

Let κ^+ be endowed with either the discrete or the order topology. We shall prove that the space $(\kappa^+)^{\kappa^+}$ is not κ-Ohio complete. From this result we will deduce that, if κ is less than the first weakly inaccessible cardinal, the space ω^{κ^+} is not κ-Ohio complete.

In relation to Question 2 it is interesting to notice that it is unknown if even the product of a κ-Ohio complete space with a compact space is κ-Ohio complete. It turns out that if this is the case then κ-Ohio completeness is closed-hereditary; so if Question 2 has a positive answer then Question 1 has a positive answer as well.

Regarding Question 1, we shall give a characterization of closed subspaces of κ-Ohio complete spaces, for uncountable cardinals κ. We do not know whether such a characterization holds for the countable case.

On spectral spaces

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A topological space is called spectral if it is homeomorphic to the prime spectrum of a ring equipped with Zariski topology. We give necessary and sufficient conditions on the space X in order to get its one point compactification (resp., Wallman compactification) spectral. We also deal with topological properties of a space such that its T0-compactification (resp., prime closed compactification) is spectral.

Alexander-Spanier Cohomology Groups of Mapping Cone

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Alexander-Spanier type cohomology groups of mapping cone C_f of continuous function $f : X \longrightarrow Y$ are studied. There is constructed an isomorphism of cohomology groups induced by cochain map $h : C^*(C_f; G) \longrightarrow C^*(f^\#; G)$ from the Alexander-Spanier cochain complex of cone C_f to the algebraic cochain cone $C^*(f^\#; G)$ of cochain map $f^\# : C^*(Y; G) \longrightarrow C^*(X; G)$. Using the obtained results, the Alexander-Spanier normal cohomology functors are investigated [1].

References

[1] V. Baladze and A. Beridze, *On Alexander-Spanier normal cohomology groups*, to appear

Measurable selectors, proximinality and integration of multi-functions

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Coauthors: V. Kadets, J. Rodriguez and M. Raja

Kuratowski and Ryll-Nardzewski's theorem about the existence of measurable selectors for multi-functions is a magnificent tool to obtain measurable selectors of suitable multi-functions; one of the drawbacks of this result is that separability is required for the range space. In this lecture we will show how to use, in some cases, descriptive set-theoretic techniques to overcome the above separability assumption and use Kuratowski and Ryll-Nardzewski's theorem to deduce that $L^1(\mu, Y)$ is proximinal in $L^1(\mu, X)$ when $Y \subset X$ is a proximinal subspace and the Banach space X is *nice*, for instance WCG. Starting again from Kuratowski and Ryll-Nardzewski's theorem and inspired by topological results involving σ-fragmented multi-functions and Baire one functions, we will present our advances when studying the existence of measurable selectors for multi-functions whose values are weakly compact subsets of a Banach space *without separability* assumptions about the range space: on one hand, we characterize multi-functions having strongly measurable selectors; on the other hand, we prove that every scalarly measurable multi-function admits scalarly measurable selectors. By doing so we solve an open problem in the area and extend the theory of Pettis integration for multi-functions that previously was only known in the separable case to the case of general Banach spaces. We will finish by showing the parallelism between the techniques presented and some questions arising again from topology.

Minimal Extension of a Certain Cascade with T_0-topological Phase Space and Continuous Closed Endomorphisms

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Coauthors: Petr Skorkovský

There is constructed the cascade formed by an action of the additive monoid of all non-negative integers on the phase set which is a sum of presheaves of solution spaces of homogeneous ordinary second-order differential equations of the form $y'' + p(x)y' + q(x)y = 0$, where pairs of functions $[p, q]$ are running through $C(J) \times C(J)$ (the set of all pairs of continuous functions defined on some interval J of reals). If any presheaf is extended by a countable chain then there exists infinitely many T_0-topologies T on the new phase set ETA of the obtained cascade such that its endomorphism monoid coincides with the monoid of all continuous closed transformations of the space (ETA, T).

On a Subset System-based Generalization of Topology

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For a quadruple S of subset systems, the aim of this talk is to introduce S-systems generalizing topologies, co-topologies, pretopologies, closure systems and kernel systems. We will mainly focus on some elementary relations between S-systems and ordered structures in the level of categories.

Interior and Closure Operators on Texture Spaces

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This talk is devoted to a discussion of closure operators for the theory of texture spaces in the sense of [2]. The recent works on fuzzy closure spaces can be found in [4,6,7]. In particular, a generalization of L-closure spaces and the natural counterparts of the Lowen functors w_L and i_L are studied in a fixed-basis setting in [7] and here the category L-CLOSURE is defined considering the closure operators on L^X and the morphisms as Zadeh type powerset operators between the objects for a fixed Hutton algebra L. Recall that Hutton algebras and the mappings preserving the arbitrary meets, joins and involution form a category which is denoted by $HutAlg$, and $HutAlg^{op}$ and $Fuzlat$ are equivalent categories [3,5]. Essentially, here we consider the closure operator on a Hutton algebra L and in a natural way, we define the category HCL of Hutton closure spaces taking the morphisms of the category $HutAlg^{op}$ with the corresponding continuity condition. In this case, the categories L-CLOSURE and H can be considered as a subcategory and a full subcategory of HCL, respectively – H denotes the category of Hutton spaces [3]. It is known that H has products and sums [1,3]. More generally, we prove that the category HCL has also products and sums. Finally, we show that the functor w_L can be also given in a textural framework for $L = [0, 1]$.

References

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[4] A.S. Mashhour, M.H. Ghanim, *Fuzzy closure space*, J. Math. Anal. Appl. vol. **106** (1985) 154–170

[5] S. E. Rodabaugh, *Point-set lattice-theoretic topology*, Fuzzy Sets and Systems, vol. **40** (1991) 297–345

[6] R. Srivastava, A.K. Srivastava, A. Choubey, *Fuzzy closure spaces*, J. Fuzzy Math. vol. **2** (1994) 525–534

[7] Wu-Neng Zhou, *Generalization of L-closure spaces*, Fuzzy Sets and Systems, vol. **149** (2005) 415–432

A de Vries-type duality theorem for locally compact spaces

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A duality theorem for the category of locally compact Hausdorff spaces and continuous maps which generalizes the well-known Duality Theorem of de Vries will be presented. Some applications will be given. The main between them are the following: (a) a slight generalization of the Stone's results concerning the extension of his Duality Theorem to the category of locally compact zero-dimensional Hausdorff spaces; (b) a completion theorem for local contact Boolean algebras; (c) a description of the spaces which are co-absolute with the (zero-dimensional) Eberlein compacts.

Preimage-wise convergences and group topologies coarser than the Isbell topology

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Convergences and topologies on functions spaces defined with the aid of collections of compact families on the underlying spaces are studied. Pointwise convergence, compact-open topology, Isbell topology and the natural convergence are instances. They are characterized setwise in terms of the corresponding convergences and topologies on hyperspaces. Transfer of properties between function spaces, hyperspaces and underlying spaces are established. This transfer hinges on the continuity of translations in the functional space. Pointwise convergence, compact-open topology and the natural convergence are translation-invariant, but the Isbell topology is not in general. We study conditions under which the Isbell topology is translation-invariant (group topology). We construct a maximal hereditary collection of compact families, for which the functional space is a group (equivalently, a topological vector space) included in the Isbell topology. The Isbell topology coincides with this vector space topology if and only if the underlying space is infraconsonant. Examples based on measure theoretic methods, show that the mentioned vector topology can be strictly finer than the compact-open topology.

Causpaces

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Alexandroff spaces ("Diskrete Räume") of Alexandroff possess a structure suitable for approximating bounded portions of physically important manifolds. Also, there has recently been some interest from physicists in using category theory to model quantum physics.

This paper is devoted to topological and categorical studies of some typical Alexandroff spaces, hoping that this will be of interest in Theoretical Physics.

Let X be a set and $f : X \longrightarrow X$ be a mapping. We denote by $\mathcal{A}(f)$ the Alexandroff topology defined by the Kuratowski closure $\mu_f : 2^X \longrightarrow 2^X$ such that $\mu_f(L) = \bigcup_{n\in\mathbb{N}} f^n(L)$.

For A topological space X, and $x, y \in X$ such that $y \in \overline{\{x\}}$, we set

$$[x, y] := \{\ z \in X \mid \overline{\{y\}} \subseteq \overline{\{z\}} \subseteq \overline{\{x\}}\ \}.$$

We say that X is a *causpace* if whenever $y \in \overline{\{x\}}$, the set $[x, y]$ is finite.

By a $\mathcal{J}$*-space*, we mean an Alexandroff topological space X satisfying the following properties:

(i) X is a causpace.

(ii) If $[x, x] \neq \{x\}$, then $[x, x] = \overline{\{x\}}$.

(iii) If $[x, x] = \{x\}$ and x is not closed, then $\overline{\{x\}} \setminus \{x\}$ has a generic point (i.e., there exists $y \in \overline{\{x\}} \setminus \{x\}$ such that $\overline{\{x\}} \setminus \{x\} = \overline{\{y\}}$).

In this paper, we prove that $(X, \mathcal{T})$ is a $\mathcal{J}$-space if and only if there is a map $f : X \longrightarrow X$ such that $\mathcal{T} = \mathcal{A}(f)$.

For convenience, let us call an *endomorphism* each pair (X, f), where X is a set and $f : X \longrightarrow X$ is a map. We let **End** be the category whose objects are endomorphisms; and an arrow (called *endo-arrow*) from (X, f) into (Y, g) is a map $\varphi : X \longrightarrow Y$ such that $g \circ \varphi = \varphi \circ f$.

Let $f : X \longrightarrow X$ be a map. We say that (X, f) is a $\mathcal{J}$*-endomorphism* if $Fix(f) = Fix(f^n)$, for each $n \in \mathbb{N}$, where $Fix(f)$ is the set of all fixed points of f. We let **JEnd** be the full subcategory of **End** whose objects are $\mathcal{J}$-endomorphisms. We prove that **JEnd** is a reflective subcategory of **End**.

Let us denote by $\mathbf{JTop_0}$ the category whose objects are $\mathcal{J}$-spaces satisfying the separation axiom T_0 and arrows are injective closed continuous maps. Then we show that $\mathbf{JTop_0}$ is isomorphic to **JEnd** by changing arrows in **JEnd** into injective endo-arrows.

$C(\tau)$-cosmic spaces

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In this paper we introduce and study the notion of $C(\tau)$-cosmic space, where τ is an infinite cardinal. Particularly, we prove that in the class of all $C(\tau)$-cosmic spaces there exists universal element.

Work supported by the Caratheodory Programme of the University of Patras.

Some results on Best Approximation in Fuzzy Metric Spaces

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Veeramani studied best approximation problem in fuzzy metric spaces based on a notion of fuzzy metric spaces introduced by George and Veeramani. In this paper we are going to prove some topological theorems and generalizes some definitions about t-best approximation.

On convergence in fuzzy metric spaces

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Since the class of fuzzy metrics in the sense of George and Veeramani includes in its definition a parameter t, it allows to introduce novel (fuzzy metric) concepts with respect to the classical metric concepts. In this sense, D. Mihet modified the definition of convergence and obtained a more general concept which is called p-convergence. In this talk, we characterize those fuzzy metric spaces, that we call *principal*, in which both concepts agree. Later, we introduce and study a concept of p-Cauchy sequence. Some illustrative examples, including a non-principal fuzzy metric space, which is not completable, are given.

On Fuzzy Λ_b Sets and Fuzzy Λ_b Continuity

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The aim of this talk is to introduce a new class of fuzzy open sets called fuzzy Λ_b sets which includes the class of fuzzy γ-open sets due to Hanafy [I. M. Hanafy, Fuzzy γ-open sets and fuzzy γ-continuity, J. Fuzzy Math. vol. 7 (1999) 419–430]. We also define a weaker form of fuzzy Λ_b sets termed as fuzzy locally Λ_b sets. By means of these new sets, we present the notions of fuzzy Λ_b continuity and fuzzy locally Λ_b continuity which are weaker than fuzzy γ-continuity due to Hanafy and furthermore we investigate the relationships between these new types of continuity and some others.

Topological Algebraic Structure on $\mathbb{R}$ with The Density Topology

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Coauthors: R.W. Heath and Thomas J. Poerio

The density of a subset E of $\mathbb{R}$ at a point x is defined to be the limit, as h goes to 0, of $\frac{m_1(E\cap(x-h,x+h))}{2h}$, where m_l is Lebesgue measure. In the density topology a set is open if the density of the set is 1 at each of its points. Tall [Pacific Journal, 1976] showed that a subset of R is connected in the density topology iff it is connected in the open interval topology. In his dissertation Poerio used Hugo Steinhaus's theorem (Fundamenta, 1920) to show that neither $\{\mathbb{R},+\}$ nor $\{\mathbb{R}^+,\times\}$ can be a cancellative topological semigroup in the density topology. We show that there can be no topological group on $\mathbb{R}$ with the density topology and examine the general case for abstract cancellative topological semigroups on $\mathbb{R}$ with the density topology.

Preservation of completeness by some continuous maps.

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Let $f\colon X \to Y$ be a continuous mapping of a completely metrizable space X onto a metrizable space Y. If every open neighborhood G of any countable metrically discrete set D contains a set S such that $D \subset S \subset G$ and $f(S)$ is resolvable (e.g., it belongs to the algebra generated by open sets), then Y is completely metrizable.

This is a result of the paper P. Holický and R. Pol, *On a question by Alexey Ostrovsky concerning preservation of completeness*, which was submitted for publication.

We shall present this result and some related ones, e.g., on Čech complete spaces X.

On Minimal Semi-Open Sets

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We introduce and discuss minimal semi-open sets in topological spaces which generalize minimal open sets defined and discussed by F. Nakaoka and N. Oda [3]. We establish some basic properties of minimal semi-open sets .We obtain some properties of pre semi-open sets using properties of minimal semi-open sets. As an application of a theory of minimal semi-open sets, we obtain a sufficient condition for a semi-locally finite space to be a pre semi-Hausdorff space

Some problems on the base dimension I

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Let X be a completely regular space and C a normal base for the closed subsets of X. The base dimension I of X by C is an element $\mathrm{I}(X, C)$ of the class $\mathcal{O} \cup \{-1, \infty\}$, where $\mathcal{O}$ is the class of all ordinals, defined by induction by the following conditions:

(1) $\mathrm{I}(X, C) = -1$ if and only if $X = \emptyset$.

(2) $\mathrm{I}(X, C) \leq \alpha$, where α is an ordinal, if and only if for every pair (F, G) of disjoint elements of C there exists a screening (L, H) of (F, G) such that $\mathrm{I}(L \cap H, C|_{L \cap H}) < \alpha$, where $C|_{L \cap H} = \{(L \cap H) \cap F : F \in C\}$.

Therefore, $\mathrm{I}(X, C) = \infty$ if and only if the relation $\mathrm{I}(X, C) \leq \alpha$ is not true for every ordinal α.

We shall consider some properties of the base dimension I and put some problems. The base dimension I is studied in the following works:

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On some categories arising in the theory of locally compact extensions

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We give a direct proof of the fact that the following three categories are isomorphic: the category of separated local proximity spaces and equicontinuous mappings, the category of LC-proximity spaces and SR-proximally continuous functions, and the category of separated L-supertopological spaces and supertopological mappings. Many basic statements of the theory of Efremovich proximity spaces are generalized for the class of local proximity spaces.

Symmetric Topological Spaces and Lattice Equivalence

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In this paper, we characterize Symmetric spaces (or R_0-spaces) by lattice equivalence. We show that a topological space X is Symmetric if and only if X and $T_1(X)$ are lattice equivalence. It is also proved that R_0 is not a lattice-invariant property, but the property "R_0 and quasi sober " is a lattice -invariant property.

On covering homomorphisms

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Let Y be a connected group and let $f : X \to Y$ be a covering mapping from a connected space X. We say that f is a covering homomorphism if it is possible to define a multiplication $\cdot$ on X in such a way that X becomes a topological group and f becomes a homomorphism of topological groups. It is well-known that each covering mapping $f : X \to Y$ from a pathwise connected space X on a pathwise connected, locally pathwise connected group Y is a covering homomorphism. In 1972, R.H. Fox introduced a notion of on overlay mapping in order to extend

the classical classification theorem for covering mappings to arbitrary connected metric spaces. Using shape-theoretic techniques we show that a covering mapping $f : X \to Y$ on a compact connected group Y is a covering homomorphism if and only if f is an overlay mapping. In particular, each finite-sheeted covering mapping $f : X \to Y$ on a compact connected group Y is a covering homomorphism.

Continuous Homology

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J. Milnor [1] on the category A_C of compact pairs (X, A) defined the homology H_*^M and proved that if (X, A) is compact metrical pair, then his homology is isomorphic to the Steenrod homology. We define continuous homology h_* and prove that if (X, A) is compact pair and coefficients group is the topological abelian group S^1 – one-dimensional sphere, then there is an isomorphism $h_*(X, A, S^1) \approx H_*^M(X, A, R)$, where R is the topological abelian group of real numbers.

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On some positional dimension-like functions

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In [1] some positional dimension-like functions are defined. These functions were studied only with respect to the property of universality. Here, we first compare and then study these functions with respect to other standard properties of dimension theory (subspace, product, and sum theorems).

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Examples of fuzzy metrics and applications

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Fuzzy metrics in the sense of George and Veeramani, are interesting for Engineering problems because of their usefulness within fuzzy systems. In this talk, we provide a series of new examples of fuzzy metrics which are interesting because of their fuzzy metric properties. As an example of engineering application, we use these fuzzy metrics for color image filtering by means of a vector ordering approach. We show that the obtained results are promising.

Folding and fundamental group of the dual graphs

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In this paper the folding of the planar graphs is discussed. Relations between the folding of the graph and the duality are deduced. The fundamental group of the dual graph is introduced. Theorems governing these relations are achieved.

On Lindeöf spaces of continuous functions

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A few theorems about the Lindelöf property of spaces of continuous functions in the topology of pointwise convergence on a Tychonoff space and on its subspaces.

Set-open topology

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We study set-open topology on the set of all continuous real-valued functions on a Tychonoff space in a general setting and compare this topology with several well-known and lesser known topologies.

Metrizable images of the Sorgenfrey line

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Recall that the Sorgenfrey line $\mathbb{S}$ is the real line topologized by a basis of half-open intervals closed on the left. We study the following question: what are metrizable images of $\mathbb{S}$ under different kinds of continuous maps.

In 1984 D. B. Motorov proved [1] that metrizable images of $\mathbb{S}$ are exactly the A-sets. In 1988 S. A. Svetlichnyj proved [2] that every open metrizable image of $\mathbb{S}$ is polish space.

We give the characterizations of metrizable images of the Sorgenfrey line under open maps, under closed maps, under closed-and-open maps, under quotient maps and under one-to-one maps.

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On hereditarily indecomposable partitions in cylinders over continua

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J. Krasinkiewicz asked in [Krasinkiewicz, On approximation of mappings into 1-manifolds, Bull. Pol. Acad. Sci. 44 (1996), 431-440] whether for every metrizable continuum X there exists a partition L between the top and the bottom of the cylinder $X \times I$ such that L is a hereditarily indecomposable continuum. We answer this question in the negative. We also discuss the class of all continua X satisfying the condition described above.

Extension Theory and the First Uncountable Ordinal Space

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Let $[0, \Omega)$ denote the first uncountable ordinal space. Suppose that K is a CW-complex, Z is a compact metrizable space, and K is an absolute extensor for Z. This means that for each closed subset A of Z and map $f : A \to K$, there exists a map $F : Z \to K$ that extends f. This is the fundamental notion of extension theory. It allows one to unify the theories of covering dimension dim and cohomological dimension $\dim_G$ modulo an abelian group G by varying the CW-complex K.

Let $Y = Z \times [0, \Omega)$. We shall discuss our result that K is an absolute extensor both for Y and $\beta(Y)$. Using $K = S^n$, one may conclude that if $\dim Z \leq n$, then the same is true of both Y and $\beta(Y)$. A parallel result is true for $\dim_G$ when one puts $K = K(G, n)$, the latter being an Eilenberg-MacLane CW-complex of type (G, n). That is, if $\dim_G Z \leq n$, then both $\dim_G Y \leq n$ and $\dim_G \beta(Y) \leq n$.

Menger subsets of the Sorgenfrey line

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Lelek noted in 1964 that, if L is a Lusin set in the real line, then the space L with the subspace topology of the Sorgenfrey line has the Menger property. We further investigate Menger subsets of the Sorgenfrey line. We give a sufficient or necessary condition for a subset of the Sorgenfrey line to have the Menger property.

Fixed point theorems and $(n-)$continuous L^*-operators

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An L^*-operator on a topological space X is a function $\Lambda : [X]^{<\omega} \to 2^X$ satisfying the following condition: If $A \in [X]^{<\omega}$ and $\{U_x : x \in A\}$ is an open cover of X, then there exists a non-empty $B \subseteq A$ such that $\Lambda(B) \cap \bigcap \{U_x : x \in B\}$ is non-empty. An L^*-operator Λ on a topological space X is said to be $(n-)$ *continuous* if for each point $p \in X$ and each neighborhood U of p there exists a neighborhood V of p such that $\Lambda(A) \subseteq U$ for each $A \in [V]^{<\omega}$ (for each $A \in [V]^{\leq n+1}$). We are going to present some fixed point and equilibrium theorems on topological spaces that admit $(n-)$ *continuous* L^*-operators. We will also give an example of an L^*-operator on a metric space which is $n -$ *continuous* for each $n \geq 1$ and not continuous.

Essentially Pseudoradial Spaces

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The notion of essential sequence and essential space is given within the class of pseudoradial spaces. If κ is a regular cardinal, we say that a κ-sequence $\langle x_\alpha \rangle_{\alpha<\kappa}$ in a topological space is *essential* if it is injective, converging and $\overline{\{x_\alpha : \alpha < \kappa\}} = \{x_\alpha : \alpha < \kappa\} \cup \{x\}$, where $x := \lim x_\alpha$

is different from all x_α. This concept, by which we try to clarify the behaviour of convergent (long) sequences, is confronted with the concepts of thin and free sequence and shown independent of them. Various examples and implications are proved among classes of Whyburn, weakly Whyburn and essentially pseudoradial (radial, semiradial, almost radial) spaces. It is shown that essentially radial spaces are the same as radial and Whyburn spaces. Some results are given concerning the existence of converging long sequences in topological spaces where no usual converging sequence exist.

On Some Closed Sets in Ideal Minimal Spaces

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The aim of this talk is to introduce ideal minimal space and to investigate the relationships between minimal space and ideal minimal space. We define some closed sets in these spaces to establish their relationships. Basic properties and characterizations related to these sets are given.

On almost GP-spaces

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A T_1 topological space X is called an almost GP-space if every dense G_δ-set of X has nonempty interior. The behaviour of almost GP-spaces under taking subspaces and superspaces, images and preimages and products is studied. If each dense G_δ-set of an almost GP-space X has dense interior in X, then X is called a GID-space. In this paper, some interesting properties of GID-spaces are investigated. We will generalize some theorems that hold in almost P-spaces.

Set-Theoretic Topology

The limits of convergent sequences in Bell's compactification

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We regard a compactification $\beta\mathbb{N}$ of a countable discrete space $\mathbb{N}$ with ccc non-separable remainder, constructed by M. G. Bell. We prove the necessary and sufficient conditions for the point $x \in \beta\mathbb{N} \setminus \mathbb{N}$ be a limit of convergent sequence of points from $\mathbb{N} \subset \beta\mathbb{N}$.

Regularity properties on the second level of the projective hierarchy

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Regularity properties of sets of reals, like Lebesgue measurability, the Baire property, or the Ramsey property, typically hold for analytic and and coanalytic sets, while they usually fail in the constructible universe L for $\mathbf{\Delta}^1_2$-sets. In fact, regularity properties for sets on the second level of the projective hierarchy (i.e. for $\mathbf{\Delta}^1_2$ and $\mathbf{\Sigma}^1_2$-sets) can often be characterized as transcendence statements over L. For example, the Baire property for $\mathbf{\Delta}^1_2$-sets is equivalent to saying that there is a Cohen real over every $L[x]$. Such characterizations are very useful for establishing implications and non-implications between various regularity properties on the second level of the projective hierarchy.

In this talk, I will give a brief survey of this area of research, and then touch upon some recent results on

- the Baire property in the eventually different topology on the second level of the projective hierarchy (joint work with Benedikt Löwe),
- polarized partition properties on the second level of the projective hierarchy (joint work with Yurii Khomskii).

Some notes about Katowice problem

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Katowice problem: Is it consistent with ZFC that ω^* is homeomorphic to ω_1^*? This can be equivalently stated as: Can Boolean algebra $\mathcal{P}(\omega)/Fin$ be homeomorphic to $\mathcal{P}(\omega_1)/Fin$? Some consequences of existence of such homeomorphism are $\mathfrak{d} = \omega_1$ and the existence of a strong Q-sequence of size ω_1 (also called uniformizable AD-system). There is a model of ZFC where both these consequences hold true.

Spaces which are Selectively Separable

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M. Scheepers introduced the notion of a space being selectively separable: for each countable sequence of dense subsets, it is possible to select a finite subset of each with dense union. It is also interesting to strengthen this notion by reformulating it as a two player game and asking about winning strategies. We consider real and consistent examples of spaces with or without these properties.

Homeomorphisms of Bagpipes

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A quarter of a century ago Nyikos gave in his Bagpipe Theorem a decomposition of an ω-bounded surface Σ into a union of a compact surface with finitely many holes, K, and the same number of long pipes, $P_1, \ldots P_n$. Recently I have been exploring the group of homeomorphisms of non-metrisable manifolds and especially its quotient subgroup, the mapping class group $\mathcal{M}(\Sigma)$. If $\Sigma = K \cup \left(\bigcup_{i=1}^{n} P_i\right)$ is a Nyikos decomposition of an orientable ω-bounded surface and $h : \Sigma \to \Sigma$ is an orientation-preserving homeomorphism then h is isotopic to a homeomorphism $g : \Sigma \to \Sigma$ such that $g(K) = K$ and g^m is the identity on ∂K for some positive integer m. Thus the structure of $\mathcal{M}(\Sigma)$ splits into those of the mapping class groups of homeomorphisms of compact surfaces with boundary and of long pipes. Some homeomorphisms having finite torsion will be considered.

Some properties of Bell's compactification of $\mathbb{N}$.

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We regard some properties of a compactification $\beta\mathbb{N}$ of countable discrete space $\mathbb{N}$ with ccc non-separable remainder, constructed by M. G. Bell. We prove some facts about subsets of $\mathbb{N} \subset \beta\mathbb{N}$, whose closures in $\beta\mathbb{N}$ are homeomorphic to Čech-Stone compactification of $\mathbb{N}$ and subsets of $\mathbb{N} \subset \beta\mathbb{N}$ whose closures are homeomorphic to a convergent sequence.

A concrete co-existential map that is not confluent.

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The notion of a co-existential map between compact spaces is obtained by dualizing the notion of an existential embedding from Model Theory. Because these embeddings are well-behaved one would expect their duals to be well-behaved as well. We describe a very concrete example of a co-existential map between continua that is not confluent; it is inspired by but easier to visualize than an example constructed by Paul Bankston.

On a problem of Fremlin

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A family D of finite subsets of κ is 1/2-filling if it is hereditary and every finite subset of κ contains a subset in D whose size is at least half of the size of the original set. A subset of κ is D-homogeneous if all of its finite subsets are in D.

We will discuss the following problem of D. Fremlin: For which $\lambda \leq \kappa$ is it true that every 1/2-filling family D of finite subsets of κ has a D-homogeneous set of size λ.

Interpolation of κ-compactness

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We call a topological space κ-compact if every subset of size κ has a complete accumulation point in it. Let $\Phi(\mu, \kappa, \lambda)$ denote the following statement: $\mu < \kappa < \lambda = cf(\lambda)$ and there are λ subsets of κ of size μ, say $\{S_\xi : \xi < \lambda\}$, such that $|\{\xi : |S_\xi \cap A| = \mu\}| < \lambda$ whenever A is any subset of κ of cardinality less than κ. We show that if $\Phi(\mu, \kappa, \lambda)$ holds and the space X is both μ-compact and λ-compact then X is κ-compact as well. Moreover, from PCF theory we deduce $\Phi(cf(\kappa), \kappa, \kappa^+)$ for every singular cardinal κ. As a corollary we get that a linearly Lindelöf and

$\aleph_\omega$-compact space is uncountably compact, that is κ-compact for all uncountable cardinals κ.

Galois-Tukey connections involving sets of metrics

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In our paper [Kada et al., Topology Appl. 153(2006), 3313-3319], we studied the smallest cardinality of a set of compatible metrics on a metrizable space X which enables the approximation to the Stone-Cech compactification of X by corresponding Smirnov compactifications. We will refine the results in this paper from the viewpoint of generalized Galois-Tukey connections, and investigate the connections among order structures of compatible metrics of separable metrizable spaces and other order structures.

On topologically induced b-convergences

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In 1966 Lodato raises the question whether it is possible to display a set of axioms for a binary relation on the powerset of a set X so that these postulates are being satisfied iff there exists a topological space Y in which X can be embedded such that subsets A,B are near in X iff their closures meet in Y. Then he gives an answer by introducing the concept of the later so called Lodato proximity spaces. Afterwards, in 1975, Bentley generalized this theorem to bunch-determined nearness spaces. In this connection we also recall that each topology on a set X, given by a closure operator "cl" , defines a compatible Leader proximity on it by setting B is near to A iff B meets the closure of A. In 1964 Doitchinov introduced the notion of supertopological spaces in order to construct a unified theory of topological and proximity spaces. As an application he shows that the compactly determined Hausdorff-extensions of a given topological space are closely related to a special class of supertopologies defined as b-supertopologies. But, all the above mentioned structures are special cases of the so- called b-convergence spaces, moreover uniform convergences in the sense of Preuss also can be dealt with. Consequently, we are going to solve the considered theorems in the broader realm of this new type of convergence!

Ordered topological C-(resp I-)spaces and generalized metric spaces

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An ordered topological space $(X, \mathcal{T}, \leq)$ is called a *C-space* iff $d(F)$ and $i(F)$ are closed whenever F is a closed subset of X. It is called an *I-space* iff $d(G)$ and $i(G)$ are open whenever G is an open subset of X. In this talk we discuss some results on ordered topological spaces which are inspired by the well-known Hanai-Morita-Stone(HMS) Theorem. Among others, we present a partial analogue of HMS by showing that given an ordered C-space $(X, \mathcal{T}, \leqslant)$ such that $\mathcal{T}$ is a metrizable topology then the upper and the lower topologies denoted by $\mathcal{T}^\sharp$ and $\mathcal{T}^\flat$ respectively are first countable if and only if for each $x \in X$, the boundaries of $d(x)$ and $i(x)$ are compact in $(X, \mathcal{T})$. We also introduce the notion of a $\mathcal{U}$-friendly partial order on a uniform space $(X, \mathcal{U})$, and then show that for each such uniform space the corresponding bitopological space $(X, (\mathcal{T}(\mathcal{U}))^\natural, (\mathcal{T}(\mathcal{U}))^\flat)$ is quasi-uniformizable.
[$i(x) = \{y \in X \mid y \geq x\}$, $\mathcal{T}^\sharp = \{O \in \mathcal{T} \mid O$ is an upper set $\}$, $d(x)$ and $\mathcal{T}^\flat$ are defined dually]

Cardinal sequences of scattered spaces

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Based on some ideas of Piotr Koszmider we construct first some morass-like structures, then some Delta-functions with some strong properties. Using these Delta-functions we show that it is consistent that the continuum is arbitrarily large and the family of cardinal sequences of locally compact scattered spaces contains all the sequences $\langle s_\alpha : \alpha < \omega_2 \rangle$ of cardinals satisfying $\omega \leq s_\alpha \leq 2^\omega$ for all α.

Extensions of Compactness of Tychonoff Powers of 2 in ZF

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We work in ZF, i.e., Zermelo-Fraenkel set theory without the Axiom of Choice (AC), and study the set-theoretic strength of compactness as well as extensions of compactness such as countable compactness, and compact-n, $n \in \mathbb{N}$, for Tychonoff products of the discrete space $2 = \{0, 1\}$.

A monotone version of monolithity

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This is a joint work with O. Alas and R. Wilson. We introduce monotonically monolithic and strongly monotonically monolithic spaces. It turns out that every monotonically monolithic space is a D-space; besides, some spaces have D-property precisely because they are monotonically monolithic. In particular, if X is a Lindelöf Σ-space then every subspace of $C_p(X)$ is monotonically monolithic. Strong monotone monolithity is implied by existence of a point-countable base and some classical results about spaces with point-countable base are valid for strongly monotonically monolithic spaces. We will show, among other things that every countably compact strongly monotonically monolithic space is compact and metrizable.

o-Boundedness of free topological groups

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Assuming the absence of Q-points (which is consistent with ZFC) we prove that the free topological group $F(X)$ over a Tychonov space X is o-bounded if and only if every continuous metrizable image T of X has the property $U_{fin}(O, \Omega)$ (the latter means that for every sequence $< u_n : n \in \omega >$ of open covers of T there exists a sequence $< v_n : n \in \omega >$ such that v_n is a finite subset of u_n and for every finite subset F of X there exists n with the property $F \subset \cup v_n$). This characterization gives a consistent answer to a problem posed by C. Hernandes, D. Robbie, and M. Tkachenko in 2000.

Categorial Topology

Coshape theory and applications

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The coshape theory, as a shape theory is a spectral homotopy theory. The notion of coshape of a space was introduced by T. Porter. The alternative definitions of coshape are given in the papers of A. Deliany and P. Hilton, Yu.T. Lisica, and V.Baladze. The shape and coshape functors of topological spaces, which are meaningful exstensions of homotopy functor of spaces having the homotopy type of polyhedras, CW-complexes or ANR-spaces, play important roles in geometric topology. The coshape theory is closely connected with the extensions of (co)homotopy and (co)homology functors from the category of spaces having the homotopy type of polyhedras to the category of all topological spaces. In particular, the spectral (co)homotopy groups and the spectral singular (co)homology groups of spaces are invariant functors of coshape theory. Besides, the (co)homotopy and (co)homology inj-groups and pro-groups of spaces also induce coshape invariant functors. Note that the inj-groups and pro-groups are important coshape invariants because they contain much more information about the direct and inverse systems than their limits, even if these limits exist. The problem of extension of functors from the subcategory of spaces having the homotopy type of good spaces to the category of general topological spaces is one of the important problems of algebraic topology. The achievements in the solution of this problem have interesting applications in different branches of modern topology and algebra. Our talk is devoted to coshape theory and its applications. It consists of two parts and deal to the following questions:

Part I. Coshape theory

1. Abstract coshape category.

2. Topological coshape category.
3. Coshape of map.

Part II. Applications

1. Extensions of functors.
2. Exact sequences of inj-groups of spaces and maps.
3. The relative Hurewicz theorem in coshape theory.

How does Universality of coproducts depend on their size?

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For a regular cardinal a, we give an example of a category of uniform spaces, in which coproducts of size a are not universal, but smaller ones are.

Quasi-uniformities as lax proalgebras

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In this talk we will focus on the study of quasi-uniform spaces as lax proalgebras, in the sense of [CHT]. In particular we will show that Cauchy-completeness can be viewed as (categorical) Lawvere-completeness (as detailed in [CH]) and we will generalize Salbany's completion monad [S].

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The semicontinuous quasi-uniformity of a frame revisited

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In this talk we present a new treatment of the pointfree version of the semicontinuous quasi-uniformity [1] based on the new tool of the ring of arbitrary (not necessarily continuous) real-valued functions made available recently by Gutiérrez García, Kubiak and Picado [2].

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Local metrically generated theories

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A metrically generated theory is a category X which is generated by its metrizable objects, in the sense that there exists a functor K from a category C of (generalised) metric spaces to X such that K preserves initial morphisms and that $K(C)$ is initially dense in the category. Metrically generated theories can be isomorphically described as concretely coreflective subconstructs of some model category. In this context, a metrically generated category essentially consists of sets structured by collections of C-metrics which are saturated in some sense. This allows for a unifying treatment of many theories, like topological spaces, uniform spaces, approach spaces, approach uniformities, but also bornological or measurable spaces, etc.

We know that a topology (resp. completely regular topology) can be derived from a quasi-uniformity (resp. uniformity) by some kind of 'localisation', i.e. by way of deriving neighbourhoods from entourages. In this talk I want to show that, using metrically generated theories, it is possible to translate this transition into rigorous mathematical results. Therefore, I will introduce what we call a local metrically generated theory, and then I will show that every metrically generated theory has a unique largest local theory which is contained in it.

On (co)normal closure operators

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Given a class of groups $\mathcal{A} \subseteq GRP$, the normal closure induced by $\mathcal{A}$ is given, for a subgroup H of G, by

$$\begin{aligned} norm_G^{\mathcal{A}}(H) := & \bigcap \{N | H \subseteq N \text{ normal subgroup of } G,\ G/N \subseteq B \in \mathcal{A}\} \\ = & \bigcap \{\ker f | f : G \to B \in \mathcal{A},\ f(H) = 0\}. \end{aligned}$$

It is easy to see that the normal closure can be defined in any category with a 0-object and an M-right factorization, where M contains all normal monomorphisms.

It is patent that the constructions of the normal and the regular closure are very similar. Accordingly, we will define the conormal closure – in parallel with the coregular closure – and, using a unifying setting, we will generalize results obtained for regular/coregular closures in [CT].

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'Distributors at work' in Topology

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"Analogies are useful in mathematics for generalising a class of well-known examples to a wider class of equally or even more useful structures. Category Theory is particularly well suited for this purpose which is no wonder as it has been developed for precisely this purpose" [2]. This claim is exemplified in [2] by the notion of distributor as "a generalisation of relations between sets to relations between (small) categories".

Already F. Hausdorff observed the similarity between the transitivity law of an ordered set and the triangle inequality of a metric space, and both can be seen as an instance of the composition law of a category [3]. Thanks to M. Barr [1] we know that topological spaces can be presented as categories (or ordered sets, if you prefer) as well, by interpreting the convergence relation $\mathfrak{x} \to x$ between ultrafilters and points of a topological space X as arrows in X. In this talk we will built on this analogy and consequently use concepts and results like

- distributor, adjunction, dual space and the Yoneda lemma

in order to study properties like

- compact ordered, soberness, Cauchy-completeness, injectivity, ...

simultaneously in (for instance) topological, approach and (probabilistic) metric spaces. We find it remarkable that all above-mentioned properties appear as generalisations of one simple concept: existence of suprema in an ordered set.

Finally, our techniques seem to be particularly well-suited for the study of domain-theoretic notions in quantitative settings. In particular, we introduce a concept of a relative continuous V-category and develop its basic properties. This way we recover many of the well-known classical structures like continuous domains, completely distributive complete lattices and Cauchy-complete metric spaces, but there remain many more settings where the meaning of continuity is still to be explored.

The talk is based on joint work with Maria Manuel Clementino, Walter Tholen and Paweł Waszkiewicz. For more informations, please consult http://www.mat.ua.pt/pessoais/dirk.

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Embedding locales of prescribed weight into localic products

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The localic analogue of the concept of separating points from closed sets is exhibited and used to provide a localic version of the topological embedding theorem (also called the diagonal theorem). Our embedding theorem allows controlling the amount of factors of the target localic product which depends on the weight of the embeddable locale. With the product of copies of the localic unit interval, it becomes the Johnstone-Tychonoff embedding theorem for completely regular locales enriched with the cardinality ingredient, i.e. stated in terms of a universal locale.

Homotopical properties of weakly globular models of homotopy types.

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Homotopy n-types are an important class of topological spaces: they amount to CW complexes whose homotopy groups vanish in dimension higher than n. The problem of modelling homotopy types is relevant both in higher category theory and homotopy theory and received contributions from both areas. There is a particularly simple model of homotopy types in the path connected case, consisting of n-fold categories

internal to groups, also called cat^n-groups. This model, however, has the disadvantage that is it does not have an algebraic description of the Postnikov decomposition nor it is easy to establish algebraically when a map of cat^n-groups is a weak equivalence. In this talk we introduce a new model of connected n-types through a subcategory of cat^n-groups, which we call weakly globular, for which the above issues are resolved in transparent way. We also describe other homotopical properties of this model, and discuss the relevance of these structures for higher category theory.

Maximal decomposition of the Turaev-Viro HQFT

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In a previous work, I have defined the Turaev-Viro HQFT using a group associated to a spherical category. This group is called the graduator of the category and defined a graduation on the category. I will extend the construction of the Turaev-Viro HQFT for every graduation on a spherical category. Furthermore I will show that the HQFT obtained is induced by the HQFT obtained from the graduator. To obtain this result I will define an homotopical invariant for every graduation and I will compare this invariant to the homotopical Turaev-Viro invariant which is defined for the graduator.

Connectedness, disconnectedness, and light factorization structures with applications to fuzzy topology

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After a short survey on the historical development connectedness, disconnectedness, and light factorization structures are studied first in the framework of topological constructs with hereditary quotients. In this context internal characterization of connectedness and disconnectedness classes are available and the full subconstruct of totally disconnected objects can be seen as a certain reflective hull. Then these results are applied to the strong topological universe FPUConv of fuzzy preuniform convergence spaces which has been introduced earlier by the author in the realm of non-symmetric fuzzy convenient topology using fuzzy filters in the sense of P. Eklund and W. Ghler. In this fuzzy setting a product theorem for the investigated connectedness concept can be proved and there is a proper class of light factorization structures on FPUConv. This is highly remarkable since on the construct FTop of fuzzy topological spaces which can be embedded into FPUConv there are no light factorization structures.

Locally presentable topology

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A well known deficiency of the category of topological spaces is that it is not cartesian closed. Another and less known drawback is that it is not locally presentable, which is caused by the existence of a proper class of ultrafilters giving too many types of convergence. This is the main reason why, in homotopy theory, simplicial sets substitute topological spaces. We will show that the final closure of a small full subcategory of topological spaces is always locally presentable. Thus topological spaces generated by simplices can play the same role in homotopy theory as simplicial sets. We will also mention benefits of finding a locally presentable (and cofibrantly generated model) for a given homotopy category.

Initial lax extensions

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This talk contributes to the algebraization of topology via the theory of monads and lax monads and their associated algebras. We construct a monad $\boldsymbol{\mathcal{P}}$, a full lax extension $\boldsymbol{\mathcal{P}}'$ and monad morphisms into $\boldsymbol{\mathcal{P}}$ from the most important monads, as there are the identity monad, the ultrafilter monad and a monad introduced to obtain metric spaces, such that their lax extensions and their associated categories of lax algebras can be derived from the unique extension $\boldsymbol{\mathcal{P}}'$ by initial lifts via these monad morphisms. This provides us with a completely unified way to obtain

the categories Top, Ap, Met and Ord without the necessity to leave the realm of Rel as was previously required. At last, we will describe the Eilenberg-Moore algebras of these monads as well.

Hausdorff and Gromov 'distances' for some topological categories

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The Hausdorff metric for closed subsets of a compact metric space and the Gromov metric for compact metric spaces lead us to define these concepts in the context of quantale-enriched categories, as suggested previously in general terms by F. W. Lawvere who, by interpreting $d(x, y)$ as $hom(x, y)$, considered individual metric spaces as categories enriched over the (extended non-negative) real line [3]. We present the Hausdorff construction as part of a monad H on V-Cat which, when the quantale V is given by the reals (with addition as tensor product), is just the topological category of (generalized) metric spaces. Our treatment of the Gromov distance for V-categories takes advantage of the fact that H may be extended to a lax functor that may be applied not just to V-functors (= non-expansive maps in case of the prototypical V) but to so-called V-modules (= metric-compatible relations for the particular V).

This work may largely be found in [1], and we also refer to previous [4] and subsequent [5] categorical treatments of the Hausdorff metric. Time permitting we will also discuss current efforts to expand these concepts and results to (T,V)-Cat, where T is a Set-monad suitably compatible with V (see [2]), in particular to Top = (beta,2)-Cat, the category of topological spaces.

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Uniformizable and realcompact bornological universes

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A bornological universe is a topological space endowed with a bornology. Each uniform space has a natural underlying bornological universe, i.e. the underlying topological spaces endowed with the bornology of sets that are bounded in the sense of Bourbaki. A bornological universe that can be obtained in this way will be called uniformizable. We want to answer two questions: 1. what are necessary and sufficient condition under which a bornological universe is uniformizable and 2. when is such a uniformizable bornological universe isomorphic to a closed subspace of a product of real lines?

Topology in Computer Science

Duality Theory in Logic

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I will discuss duality theory and its use in non-classical logics such as intuitionistic and modal logics. The ideas go back to Stone and Tarski, and predate the birth of pointfree topology. As we will see, some of the questions that come from logic shed a new light on the known categories of topological spaces, as well as provide a new source of problems in topology, some of which still remain open.

Arcs and Curves in $\mathbb{Z}^2$

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We prove a Jordan Curve Theorem in $\mathbb{Z}^2$ equipped with the Khalimsky Topology. This proof is different from the O. Kisselmann's one (*Digital Jordan Curve Theorems*, Lecture Notes in Computer Science, Springer, Berlin , vol. 1953, (2000) and the J.Slapal's one (*Digital Jordan Curves*, Topology and Applications. 153 (2006), 3255–3264.). We use essentially the specific properties of Alexandroff spaces:

- (X, T) is an A-space if and only if there exists a binary relation R such that T is the R-right-topology.
- The connectivity in an A-space is equivalent to the COTS-Arc-connectivity.
- If X and Y are two A-spaces and f a function from X to Y, f is a continuous function if and only if it's an increasing function, X and Y are equipped with the the orders determined by the topologies in X and Y.

Arcs and curves are defined as in the plane $\mathbb{R}^2$. This result is published in AGT vol. **9,2** (2008), pp: 253-262.

Partial Metrics and Fuzzy Equalities

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Partial metrics arise in the context of programming language semantics and generalized metrization of non-Hausdorff topologies. Fuzzy equalities arise in the context of sheaves and fuzzy sets. It turns out that partial metrics and fuzzy equalities coincide.

More specifically, the axioms for partial metrics with values in quantales coincide modulo notation with the axioms given by U.Hoehle for Q-sets (M-valued sets, sets with fuzzy equality, quantale-valued sets) for the case of commutative quantales.

Omega-sets (sets valued in complete Heyting algebras) correspond to the case of partial ultrametrics.

On some closure operations on the product spaces of the Khalimsky line

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The Khalimsky topology is known for its utility for representing geometric and topological properties of the digital images in the computer science-motivated topology. In this work we investigate some relationships between three certain (not necessarily topological) closure operations defined on a product or a function space of the Khalimsky line. We consider a certain class of subspaces of such function spaces and products, for which we characterize several low separation axioms and several covering properties (including compactness) in terms of the studied closure operations.

Digital fundamental group of a digital product

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In this talk we propose a condition of which the multiplicative property of the digital fundamental group holds. Precisely, using the L_{HS}- or L_{HC}-property of the digital product with k-adjacency $(X_1 \times X_2, k)$, a k-homotopic thinning of the digital product, and various properties from digital covering and digital homotopy theories, we provide a method of calculating the k-fundamental group of the digital product.

On B-completeness and quietness of quasi-uniform spaces.

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In our previous work we have successfully extended the Doitchinov completion theory of balanced quasi-metric to arbitrary T_0-quasi-metric spaces. The resulting completion was called the B-completion.
In this talk we try to extend Doitchinov's completion theory of quiet quasi-uniform space to general T_0-quasi-uniform spaces. We only partially successful because investigation due to Deák indicates that no suitable concept of a quiet Cauchy filter pair exist which could replace the quasi-metric concept of balanced Cauchy filter pair in the quasi-uniform settings.
We shall work with a chosen family of quasi-pseudometric in order to obtain a general theory of the B-completion of subbasic families of quasi-pseudometrics that can be applied to the study of quasi-uniform spaces.

Finite approximation of spaces

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There is a need to store and manipulate topological spaces in computers with finite memories. It has long been known that each compact Hausdorff space is the subspace of closed points of an inverse limit of a system of finite T_0-spaces and continuous maps.

For such an approximation, properties of the inverse limit are closely linked to properties of the maps. For example, a map is called "normalizing" if inverse images of disjoint closed sets are always contained in disjoint open sets; it is "hereditarily normalizing" if its restriction to all subspaces is normalizing. The inverse limit of a system of T_0 -spaces and continuous maps is then (hereditarily) normal if and only if, the system is eventually (hereditarily) normalizing.

We also discuss the extension of maps between spaces to the inverse limits of these systems. Most of this is joint work with Richard Wilson, and some is joint work with V. V. Tkachuk.

The Topology of Causal Sites

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The notion of causality stands in the background of many modern physical theories and subdisciplines, like general relativity and quantum gravity, but it also has some importance for temporal logic, distributed computations and concurrent systems in computer science. A *causal site*, introduced by J. D. Christensen and L. Crane [1], is a modification of the concept of causal set, defined formerly by R. Sorkin et al. [6]. Recall that a causal site $(S, \sqsubseteq, \prec)$ is a set S of *regions* equipped with two binary relations $\sqsubseteq$, $\prec$, where $(S, \sqsubseteq)$ is a partial order having the binary suprema $\sqcup$ and the least element $\bot \in S$, and $(S \smallsetminus \{\bot\}, \prec)$ is a strict partial order (i.e. antireflexive and transitive), linked together by the following axioms, which are satisfied for all regions $a, b, c \in S$:

(i) $a \sqsubseteq b$ and $a \prec c$ implies $b \prec c$,

(ii) $b \sqsubseteq a$ and $c \prec a$ implies $c \prec b$,

(iii) $a \prec c$ and $b \prec c$ implies $a \sqcup b \prec c$.

(iv) There exits $b_a \in S$, called *cutting of a by b*, such that

   (a) $b_a \prec a$ and $b_a \sqsubseteq b$;

   (b) if $c \in S$, $c \prec a$ and $c \sqsubseteq b$ then $c \sqsubseteq b_a$.

We study some topological properties of causal sites by a topological reformulation of certain fundamental concepts of formal concept analysis (FCA) of B. Ganter and R. Wille [3]. For instance, it turns out that there is a canonical compact T_1 (not necessarily Hausdorff) topological space closely linked to a causal site. On the other hand, some physical motivated structures, even so simple as the Minkowski space, admits of more than one compatible causality sites, potentially leading to different topologies. Thus it is a natural question whether it is possible to select an appropriate causal site, generating the usual topology on the studied, physical motivated structure.

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The scale of a quasi-uniform space

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We discuss a method to define the concept of a scale of a quasi-uniform space and investigate some of its basic properties.

The scale of a uniform space was introduced by Bushaw [1] in order to investigate stability in topological dynamics. It was further studied by Kent [2] and many others (see e.g. [3,4]).

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Discrete Partial Mathematics

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Our goal is to inherently incorporate complexity theory into the form of partial mathematics that is now the mature state of research into partial metric spaces. This is to be achieved by first retracing the inspirational work of Dana Scott in order to define a partial metric counterpart for his $P(\omega)$ model of the untyped lambda-calculus. From here an inherently quantitative (as in partial metric) formulation of denotational semantics is developed specifically to model the non strict functional programming language Haskell. Discrete denotational semantics is now defined to be a restriction of quantitative denotational semantics that enables each partial metric distance to be interpretable as a counter. And so, a denotation of complexity theory is introduced for each and every Haskell program, yielding a substantive first case study in Discrete Partial Mathematics. Our research is then to be compared with the alternative approach of Efficiency-Oriented Languages advocated by Michel Schellekens.

Computational Complexity of Topological Logics

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Let T be a topological space. By "topological frame over T", we mean a non-empty collection of subsets of T. Now let L be a formal language whose variables are taken to range over the elements of some topological frame, and whose predicates and function symbols have (fixed) interpretations as familiar topological relations and operations. Thus, if F is a topological frame, we may non-problematically speak of the satisfaction of any L-formula by a tuple of elements of F. Derivatively, if K is a class of topological frames (not necessarily over the same topological space) we may speak of the satisfiability of an L-formula with respect to K. We call the pair (L, K) a "topological logic". The primary question arising in connection with any topological logic is: how do we recognize its satisfiable formulas? From an algorithmic point of view, we are particularly concerned with the decidability and complexity of these problems.

This talk presents a survey of recent advances in the complexity-theoretic analysis of topological logics. Of particular interest are the cases in which the class K consists of a single frame over some low-dimensional Euclidean space. We shall discuss what is known in these cases, and conclude with some challenging open problems.

A duality for bounded lattices with modal operators

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We aim to present a duality for bounded (not necessarily distributive) lattices with modal operators of possibility, necessity, sufficiency or dual sufficiency. The key elements are a generalisation of Goldblatt's duality for distributive lattices with meet- and join-preserving operators, and a simultaneous extension of Urquhart's duality for bounded lattices. We aim to achieve this by topologising the discrete dualities for bounded lattices with operators presented by Orlowska and Vakarelov.

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On the Wallman ordered compactification and the T_1-ordered property

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With the standard definitions and constructions, the Wallman ordered compactification of a T_1-ordered topological space is T_1 but not necessarily T_1-ordered. We present a new form of the T_1-ordered property called the T_1^K-ordered property. We show that the Wallman ordered compactification of a T_1^K-ordered topological space is T_1^K-ordered and present other compelling reasons to suggest that the T_1^K-ordered property is a more appropriate ordered analog of the T_1 topological property.

Hyperspaces of a Weightable Quasi-metric Space

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It is well known that both weightable quasi-metrics and the Hausdorff distance provide efficient tools in several areas of Computer Science. This fact suggests, in a natural way, the problem of when the upper and the lower Hausdorff quasi-pseudo-metrics of a weightable quasi-metric space (X, d) are weightable. Here we discuss this problem. Although the answer is negative in general, we show, however, that it is positive for several nice classes of (nonempty) subsets of X. Since the construction of these classes depends, to a large degree, on the specialization order of the quasi-metric d, we are able to apply our results to some distinguished quasi-metric models that appear in theoretical computer science and information theory, like the domain of words, the interval domain and the complexity space.

Convenient Alexandroff pretopologies on the digital plane

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We discuss several Alexandroff pretopologies on $\mathbb{Z}^2$ with respect to which some cycles in a certain natural graph with the vertex set $\mathbb{Z}^2$ are Jordan curves. We deal also with a closure operator on $\mathbb{Z}^2$ that is not a pretopology but has the same property.

Topology in Algebra

A Family of Asymmetric Ellis-Type Theorems

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Bouziad in 1996 generalized theorems of Montgomery (1936) and Ellis (1957), by proving that each Čech-complete space with a separately continuous group operation must be a topological group. We generalize this result by dropping the requirement that the spaces be Hausdorff or even T_1. Our theorems then apply to groups with "asymmetric" topologies, such as the additive group of reals with the upper topology, whose open sets are the open upper rays. We use the fact that each topological space has an associated second topology, which we call the "k-dual", and we consider cases where the bitopological space consisting of the original topology and its k-dual is a "Hausdorff k-bispace", the latter being a bitopological parallel to the topological concept of a Hausdorff k-space, but in which neither topology need be Hausdorff. Suppose a group has a topology in which the group multiplication is separately continuous. Assume also that the bitopological space described above is a Hausdorff k-bitopological space. One of our results is that if the join of the two topologies is Čech-complete, then inversion is a homeomorphism between the original space and its k-dual, and the group operation is jointly continuous with respect to both topologies. The same conclusion holds more generally if the join is assumed to be a Baire p-space, p-σ-fragmentable by a complete sequence of covers.

On the variety of Schwartz groups

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Schwartz groups have been defined to be those abelian Hausdorff groups which satisfy:

$\forall U \in \mathcal{U}(0)\ \exists V \in \mathcal{U}(0), \exists (F_n),$ a sequence of finite subsets of G : $V \subseteq 1/nU + F_n\ \forall n \in \mathbb{N}$.

We treat the question whether the variety of all Schwartz groups coincides with the variety generated by all locally–k_ω groups.

Image Partition Regularity Near Zero and Universally

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Many of the classical results of Ramsey Theory for example Schur's Theorem, van der Waerden's Theorem, Finite sums Theorem, are naturally stated in terms of image partition regularity of matrices. Many characterizations are known of image partition regularity over $\mathbb{N}$ and other subsemigroups of $(\mathbb{R}, +)$. In this presentation first we introduce the notion *image partition regularity near zero* and *image partition regularity near zero in the strong sense* for different dense subsemigroups of $\mathbb{R}$ and investigate the interrelations between them. Being motivated by the definition of *image partition regularity near zero* we also introduce another notion *universally image partition regularity.* We shall prove that for finite matrices all the notions are equivalent, but in the infinite matrices there are lots of variety. Finally we prove that universally image partition regular matrices exist in abundance. To this end the author should mention that some portion of this presentation is a joint research work with Prof. Neil Hindman.

Measurable functions as an epireflection

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Let $\mathbf{A}$ be a σ-field of subsets of X. Denote $\mathcal{M}(\mathbf{A})$ the set of all measurable functions of X into $[0,1]$. The set $\mathcal{M}(\mathbf{A})$ carries several canonical structures: pointwise partial order, pointwise sequential convergence and various pointwise (partial) algebraic operations generated by the corresponding algebraic operations on $[0,1]$. If we identify each set $A \in \mathbf{A}$ and its characteristic function χ_A, then $\mathbf{A}$ and $\mathcal{M}(\mathbf{A})$ become distinguished systems of fuzzy subsets of X: $\mathbf{A}$ is a domain of classical probability and $\mathcal{M}(\mathbf{A})$ is a domain of fuzzy probability. We study their topological properties and their mutual relationship from the viewpoint of category theory. We show that $\mathbf{A}$ and $\mathcal{M}(\mathbf{A})$ have properties analogous to compactness and the embedding of $\mathbf{A}$ into $\mathcal{M}(\mathbf{A})$ can be viewed as an epireflection having interesting probabilistic aspects.

Compact-like topological groups with small compact subsets

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It has long been known that some precompact topological groups admit no infinite compact subsets. Reporting on recent joint results with S. Macario and with L. Recoder and M.G. Tkachenko we will describe examples of pseudocompact groups with no infinite compact subsets and of ω-bounded groups whose compact subsets are all metrizable. We will explore as well the applications of these results in the dulaity theory of topological groups, such as the existence of Pontryagin reflexive P-groups and of Pontryagin reflexive ω-bounded groups.

Small quasi-convex sets in locally compact abelian groups

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For an abelian topological group G, we denote by $\widehat{G}$ the *Pontryagin dual* of G, that is, the group of all characters of G endowed with the compact-open topology. For $E \subseteq G$ and $A \subseteq \widehat{G}$, the *polars* of E and A are defined as $E^{\triangleright} = \{\chi \in \widehat{G} \mid \chi(E) \subseteq \mathbb{T}_+\}$ and $A^{\triangleleft} = \{x \in G \mid \forall \chi \in A, \chi(x) \in \mathbb{T}_+\}$. The set E is said to be *quasi-convex* if $E = E^{\triangleright\triangleleft}$.

Unlike the "geometrically" transparent property of convexity, quasi-convexity remains an admittedly mysterious property. Although locally quasi-convex groups have been studied by many authors (cf. [1], [2] and [3]), their work did not completely reveal the nature of the small quasi-convex sets. Interest in the compact quasi-convex sets stems from the theory of Mackey groups (cf. [4] and [8]).

A sequence $\{x_n\}_{n=0}^{\infty} \subseteq G$ is *quasi-convex* if $S = \{0\} \cup \{\pm x_n \mid n \in \mathbb{N}\}$ is quasi-convex in G. We say that $\{x_n\}_{n=0}^{\infty}$ is *non-trivial* if the set S is infinite, and it is a *null sequence* if $x_n \longrightarrow 0$. Lydia Aussenhofer asked for a characterization of compact abelian groups that admit a non-trivial quasi-convex null sequence. In this talk, we provide such a characterization for the larger class of *locally* compact abelian groups. Concrete examples of non-trivial quasi-convex null sequences in the groups $\mathbb{R}$, $\mathbb{R}/\mathbb{Z}$, and $\mathbb{Z}_p$ (the p-adic integers) will also be presented.

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The Følner function for Thompson's group F

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If G is a finitely generated amenable group, then we can define a function $f(n)$ to be the minimum cardinality of a $1/n$-Følner set in G. This function is the Følner function for G (its asymptotics do not depend on the choice of generating set). Gromov asked whether there are finitely presented amenable groups whose Følner function grows faster than every primitive recursive function. I will examine the case of Thompson's group F, demonstrating a lower bound for its Følner function in terms of the tower function.

Archimedean atomic Lattice effect algebras with Hausdorf interval topology

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It is well known that a Boolean algebra B is atomic iff the interval topology $\tau(i)$ on B is Hausdorff (Katětov, Sarymsakov at all). This statements no longer holds for generalizations of Boolean algebras as orhomodular lattices and MV-algebras. There is a complete atomic orthomodular lattice which $\tau(i)$ is not Hausdorff (Sarymsakov at all) and an MV-algebra M which $\tau(i)$ is Hausdorff, but M is not atomic ($M = [0,1] \subseteq \mathbb{R}$).

We study a common generalization of orthomodular lattices and MV-algebras, called lattice effect algebras. Namely we study a family $\mathcal{A}$

of Archimedean atomic lattice effect algebras with Hausdorff interval topology. We prove that every lattice effect algebra E in $\mathcal{A}$ is almost orthogonal, meaning that to every atom of E there exist only finitely many nonorthogonal atoms. Further, for Archimedean atomic lattice effect algebra E we show that: E is in $\mathcal{A}$ iff E is an order-continuous lattice iff E is compactly generated by finite elements.

Moreover, for Archimedean atomic lattice effect algebra E with finitely many bloks (maximal sub-MV-effect algebras) we can show that: E is almost orthogonal iff the MacNeille completion $MC(E)$ of E is compactly generated. We applied these results on the extension of any (o)-continuous state existing on sharp elements (or on finite and cofinite elements) of E onto whole E, and onto $MC(E)$ as well.

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Hewitt-Marczewski-Pondiczery type theorem for abelian groups and Markov's potential density

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The classical result of Hewitt-Marczewski-Pondiczery states: If τ is an infinite cardinal, I is a set such that $|I| \leq 2^\tau$, and for every $i \in I$ a space X_i has a dense subset of size $\leq \tau$, then the product $X = \prod_{i \in I} X_i$ also has a dense subset of size $\leq \tau$. We investigate the following "algebraic version" of this theorem. Let κ be an infinite cardinal and $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ be the circle group. Given a fixed subset S of an abelian group G, we attempt to find a group homomorphism $\pi : G \to \mathbb{T}^\kappa$ such that $\pi(S)$ becomes dense in $\mathbb{T}^\kappa$. Of particular interest is the special case when π can be chosen to be a monomorphism, that is, when the group G and the subgroup $\pi(G)$ of $\mathbb{T}^\kappa$ become isomorphic. Our choice of the target group is justified by the fact that every abelian group G is isomorphic to a subgroup of $\mathbb{T}^\kappa$ for a suitable κ. To ensure a closer resemblance of the Hewitt-Marczewski-Pondiczery theorem, we pay special attention to the case $\kappa = 2^\tau$ for some infinite τ by addressing the following question:

Given a subset S of an abelian group G such that $|S| \geq \tau$ and $|G| \leq 2^{2^\tau}$, does there exist a monomorphism $\pi : G \to \mathbb{T}^{2^\tau}$ such that $\pi(S)$ becomes dense in $\mathbb{T}^{2^\tau}$?

The origin of this setting can be traced back to the 1916 paper of Weyl [4]. We recall the classical Weyl's uniform distribution theorem: Given a faithfully indexed subset $S = \{a_n : n \in N\}$ of the integers $\mathbb{Z}$, the set of all $\alpha \in \mathbb{T}$ such that the set $S\alpha = \{a_n\alpha : n \in N\} \subseteq \mathbb{T}$ is uniformly distributed has full measure 1. Since uniform distribution implies density in $\mathbb{T}$, it follows that $S\alpha$ is dense in $\mathbb{T}$ for almost all $\alpha \in \mathbb{T}$. Every $\alpha \in \mathbb{T}$ determines uniquely a homomorphism $h_\alpha : \mathbb{Z} \to \mathbb{T}$ such that $h_\alpha(1) = \alpha$. Furthermore, $\alpha \in \mathbb{T}$ generates a dense subgroup of $\mathbb{T}$ iff α is non-torsion iff the homomorphism h_α is a monomorphism. Hence, one can state (a consequence of) Weyl's theorem by simply saying that for every infinite subset S of $\mathbb{Z}$ there exists a monomorphism $\pi : \mathbb{Z} \to \mathbb{T}$ such that $\pi(S)$ is dense in $\mathbb{T}$. Tkachenko and Yaschenko [3] consider homomorphisms $\pi : G \to \mathbb{T}^\omega$ such that $\pi(S)$ is dense in $\mathbb{T}^\omega$, for a certain class of groups G. They use such homomorphisms as a technical tool in addressing the problem suggested first in 1946 by Markov.

According to Markov [2], a subset S of a group G is called *potentially dense* (in G) provided that G admits some Hausdorff group topology $\mathcal{T}$ such that S is dense in $(G, \mathcal{T})$. The last section of [2] is exclusively dedicated to the following problem: Which subsets of a group G are potentially dense in G? Markov proved that every infinite subset of $\mathbb{Z}$ is potentially dense in $\mathbb{Z}$. This was strengthened in by Dikranjan and Tkachenko [1] who showed that every infinite subset of $\mathbb{Z}$ is dense in some precompact metric group topology on $\mathbb{Z}$. (Apparently, the authors of [2] and [1] were unaware that both these results easily follow from Weyl's uniform distribution theorem.) Further progress was obtained by Tkachenko and Yaschenko [3] who proved the following theorem: If an abelian group G of size at most continuum is either almost torsion-free or has exponent p for some prime p, then every infinite subset of G is potentially dense in G. (According to Tkachenko and Yaschenko [3], a n abelian group G is *almost torsion-free* if $r_p(G)$ is finite for every prime p.) The authors have obtained a complete characterization of the countable potentially dense subsets of abelian groups: A countable subset S of an abelian group G is potentially dense in G if and only if $|G| \leq 2^c$ and S is Zariski dense in G. (Here c denotes the cardinality of the continuum.) We also investigate the remaining case of uncountable subsets S. In particular, we obtain a new sufficient condition that guarantees that a subset S of an abelian group G is potentially dense in G. Moreover, when this condition is satisfied, we prove that the topology $\mathcal{T}$ on G such that S is $\mathcal{T}$-dense in G can be chosen to be precompact. When G belongs to a wide class of abelian groups (for example, almost torsion-free or divisible groups) and S is uncountable, our sufficient condition turns out to be also necessary for potential density. At last but not least, our sufficient condition is rather powerful in the countable case as well: the only case that is *not* covered by our condition is when nS for a suitable non-zero integer n is contained in a finitely generated subgroup of G. It is therefore only this special case that still requires the substantially more sophisticated techniques (developed by the authors earlier) to prove potential density (in some precompact group topology).

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Weakening the topology of a Lie group by forcing a sequence to converge to zero

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A weakened Lie group is a Lie group endowed with a Hausdorff group topology τ that is weaker than the Lie topology. Such topologies arise in the study of Lie groups of transformations. If the Lie group is connected, then all possible topologies τ are completely determined by their restriction to a particular abelian subgroup, and thus the problem reduces to investigating the ways in which the topology of an abelian Lie group can be weakened while remaining Hausdorff. In the present paper, we consider metrizable topologies for $\mathbb{R}^n$ that are defined by specifying a sequence of elements of $\mathbb{R}^n$ and the rate at which it converges to zero, and we explore the effect of changing the converging sequence and/or the "rate sequence." We prove, for example, that the resulting topologies are all locally isometric, provided the rate sequence is the same, and their completions are locally isometric, as well. Since the local isometry is not, in general, a local homomorphism, the completions can have different global properties.

Topology in Geometry

Minimal Peano Curve

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A Peano curve $p(x)$ with maximum square-to-linear ratio equal to $5\frac{2}{3}$ is constructed; this ratio is smaller than that of the classical Peano–Hilbert curve, whose maximum square-to-linear ratio is 6. It is proved that this curve is a unique (up to isometry) regular diagonal Peano curve of fractal genus 9 whose maximum square-to-linear ratio is less than 6. A theory is developed that allows one to find the maximum square-to-linear ratio of a regular Peano curve on the basis of computer calculations.

The torsion of almost product and almost complex projective geometries

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We discuss almost product projective geometry and almost complex projective geometry with respect to cohomological interpretation of the curvature. The appropriate cohomology is computed by the Künneth formula from the classical Kostant's formulae. Our approach is based on an observation that well known general techniques apply, and our goal is to illustrate the power of the general parabolic geometry theory on quite explicit examples.

Characteristic classes of smooth fibrations

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I will define various characteristic classes of smooth fibrations and show how to calculate some of them.

The problem of a connectedness of groups of Weil algebra automorphisms and some consequences in geometry

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Weil algebras are real local finite-dimensional commutative unital algebras. We present that both connectedness and disconnectedness can occur for groups of automorphisms of Weil algebras in usual Euclidean topology and compare with e.g. Zariski topology and also with some results about special cases which are first of all jet groups. We show effects of connectedness / disconnectedness for a characterization of a fixed point subalgebra and, consequently, for natural geometric operations on Weil bundles, which represent a generalization of higher order velocities bundles.

Comments on Parabolic Geometries

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I shall provide a brief review of the nice class of geometric structures called Parabolic Geometries. Special attention will be devoted to some topological aspects of the homogeneous models. The talk will be of survey character.

Indices of quaternionic complexes

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I will present a procedure how to compute analytical indices of a class of elliptic complexes on quaternionic manifolds. These complexes arise as subcomplexes of the so-called curved BGG-sequences in the framework of parabolic geometries.

Symmetries on parabolic geometries

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We study parabolic geometries which can carry some symmetry at a point. We show some examples and discuss existence of symmetries in dependence on the Lie groups which define the geometry.

Topology in Functional Analysis

Continuous duality for topological groups

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For Pontryagin's group duality, in the setting of locally compact topological Abelian groups, the topology on the character group was the compact-open topology. There exist, at present, two extensions of this theory to topological groups which are not necessarily locally compact. The first, called the Pontryagin dual, continues to use the compact-open topology. The second, the continuous dual, uses the continuous convergence structure. Both coincide on locally compact topological groups but differ dramatically otherwise. The Pontryagin dual is a topological group while the continuous dual is not. On the other hand, the continuous dual is a left adjoint and enjoys many categorical properties which fail for the Pontryagin dual. These two duals are examined and compared.

Topological groupoids with locally compact fibres

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For developing an algebraic theory of functions on a topological groupoid (more precisely to define convolution that gives the algebra structure on a function space associated with G), one needs an analogue of Haar measure on locally compact groups. This analogue is a system of measures, called Haar system, subject to suitable invariance and smoothness conditions called respectively "left invariance" and "continuity". By analogy with the group case, it is usual to endow the groupoid

with a locally compact topology. But unlike the case of locally compact group, Haar system on groupoid need not exists. If G is a locally compact groupoid and R is the principal groupoid associated to G, then R can be endowed with various topologies. But these topologies are not necessarily locally compact. Therefore if the groupoid does not satisfy further hypotheses, R can not be endowed with a Haar system. The purpose of this paper is to introduce a topology on G (not necessarily locally compact) such that the fibres of G are locally compact Hausdorff subspaces and to prove that we can endow R with a similar topology. We shall modify the choice of $C_c(G)$ (the space of of complex valued continuous functions with compact support on G) and continuity condition required for a Haar system, and we shall prove that any Haar system on G can be used to construct a Haar system on R, and conversely.

Duality methods in topological abelian groups

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We report on some results around the duality theory of topological abelian groups. In particular, the properties of determined groups and the application of duality methods in the computation of some topological invariant cardinals will be mainly discussed.

Frames of multipliers in tensor products of Hilbert C*-modules

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Hilbert C^*–modules are generalizations of Hilbert spaces by allowing the inner product to take values in a C^*–algebra rather than in the field of complex numbers. The notion of frames in Hilbert spaces was introduced by R. J. Duffin and A. C. Schaeffer [Trans. Amer. Math. Soc. 72(1952)] in the context of non-harmonic Fourier series. M. Frank and D. Larson [Contemp. Math. 247(1999) and J. Operator Theory 48(2002)] generalized this notion to the situation of Hilbert C^*–modules and later

I. Raeburn and S.J. Thompson [Proc. Amer. Math. Soc. 131(2003)] introduced the notion of frames of multipliers. A multiplier of a Hilbert A-module E is an adjointable module morphism from A to E. The set $M(E)$ of all multipliers of E has a structure of Hilbert C^*-module over the multiplier algebra of A and contains E as Hilbert C^*–submodule. A standard frame of multipliers for a Hilbert A-module E is a sequence h_{nn} of multipliers of E such that $\Sigma_n \langle \xi, h_n \rangle_{M(E)} \langle h_n, \xi \rangle_{M(E)}$ converges in A for each $\xi \in E$ and there are constants $C, D > 0$ with the property that

$$C \langle \xi, \xi \rangle \leq \Sigma_n \langle \xi, h_n \rangle_{M(E)} \langle h_n, \xi \rangle_{M(E)} \leq D \langle \xi, \xi \rangle$$

for each $\xi \in E$.

Given a standard frame of multipliers h_{nn} in a Hilbert A-module E and a standard frame of multipliers t_{nn} in a Hilbert B-module F we construct a standard frame of multipliers for the external tensor product of E and F and for the inner tensor product of E and F using a C^*–module morphism Φ from A to the C^*-algebra of all adjointable module morphisms on F.

Hereditary covering properties of weak*-topologies

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We characterize, in a uniform way, certain properties of a Banach space in terms of hereditary covering properties of the weak*-topology of the dual space. Among properties characterized in this way are weak countable determinedness, weak $\mathcal{K}$-analycity and the existence of an equivalent uniformly Gâteaux smooth norm.

Three-space property for analytic metrizable locally convex spaces

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By a *three-space property* (for topological vector spaces) we understand the following: Suppose that E is a tvs and F is a closed vector subspace such that F and the quotient E/F have certain property $\mathcal{P}$. Does E have property $\mathcal{P}$? Corson used the concept of weakly compactly generated Banach spaces to show that the Lindelöf property is not a three-space property. Corson's example shows also that K-analyticity is not a three-space property but it does not cover the problem for $\mathcal{P} =$ *analytic*. The main result states the following

Theorem (1) Let E be a metrizable topological vector space containing a closed subspace F such that F and E/F are analytic. If F is complete and locally convex, then E is analytic. (2) There is a separable normed space E which is not analytic but contains a closed analytic subspace F such that E/F is a separable Banach space.

The argument used in part (2) applies to provide a large class of weakly analytic metrizable and separable Baire tvs not analytic (clearly such spaces are not locally convex!).

Envelopes of open sets related to extending holomorphic functions

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Let U be an open subset of a dual Banach space. A subset of U is called U-bounded if it is bounded and has positive distance to the complement of U. The tilde-envelope of U is the union of weak* closures of all U-bounded sets. This kind of envelope is related to extending holomorphic functions. We study this envelope and its iteration, we present a number examples and establish a connection to iterated weak* sequential closures.

Trees and finitely fibered compacta

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A compact space K is called n-fibered (finitely fibered) if it has a continuous map $f\colon K \to X$ onto a metric space, such that the fibers of f are at most n-element (finite) sets.

Every tree T is naturally a locally compact topological space. We show that its one-point compactification is 2-fibered if and only if T is $\mathbb{R}$-embeddable and $|T| \leq 2^{\aleph_0}$. We shall also discuss which trees have compactifications representable as subsets of Baire class one functions over a Polish space (i.e. Rosenthal compacta). As an application, we give an example of a Rosenthal compact K which is not a continuous image of any first countable Rosenthal compact and its Banach space of continuous functions has some negative renorming properties.

On linear continuous open surjections of the spaces $C_p(X)$

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Let $I = [0,1]$ be the closed unit segment, and X be the one-point compacticitaion of the disjoint union of the n-dimensional cubes I^n for all natural numbers n. We show that there exists a linear continuous open surjection from $C_p(I)$ onto $C_p(X)$. Open questions will be duscussed.

Compactness of generalized Helly spaces.

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Given a linearly ordered set X, the subset $H(X,[0,1])$ of $[0,1]^X$ consisting of the non-decreasing mappings $u : X \to [0,1]$ is Loeb-compact in set-theory without the Axiom of Choice ZF. Moreover, if the linear order X is complete, the dual ball of the Banach space $C(X)$ endowed with the weak* topology is Loeb-compact, and the space $C(X)$ satisfies the effective continuous Hahn-Banach property.

Topological Properties of Cone Metric Space

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The purpose of this paper is to introduce Cone metric space and discuss about topological properties of Cone metric space. In this note, we shall consider the relation between induced topology by Cone metric space and induced topology by metric space and obtain some results on them.

Some remarks about K-analyticity of groups of continuous homomorphisms

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For an Abelian locally compact group X let $X_p^\wedge$ be the group of continuous homomorphisms from X into the unit circle $\mathbb{T}$ of the complex plane endowed with the pointwise topology. It is proved that the group X is metrizable if and only if $X_p^\wedge$ is a K-analytic space if and only if X endowed with its Bohr topology $\sigma(X, X^\wedge)$ has countable tightness. This enables us to establish a large class of topological groups with countable tightness which cannot be sequential, so neither Fréchet-Urysohn.

Gul'ko, descriptive, and Gruenhage compact spaces

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We present some known and some new facts about Gul'ko, descriptive, Gruenhage, and fragmentable compact spaces. We show how they reflect the geometrical structure of corresponding Banach spaces $C(K)$. In particular, we provide a proof of a recent renorming result of R. Smith by a simple transfer of Day's norm.

About weakly Lindelöf $C(X)$ not weakly Lindelöf Σ

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Cascales, Kakol, Saxon proved that in a large class $\mathfrak{G}$ of locally convex spaces E (containing all (LM)-spaces and (DF)-spaces all (LM)-spaces (hence metrizable lcs), dual metric spaces (hence (DF)-spaces), the space of distributions $D'(\Omega)$, real analytic functions $A(\Omega)$ for open $\Omega \subset \mathbb{R}^{\mathbb{N}}$) for a locally convex space $E \in \mathfrak{G}$ the weak topology $\sigma(E, E')$ of E has countable tightness if and only if its weak dual $(E', \sigma(E', E))$ is K-analytic. Applying examples of Pol (and Kunen) one gets however that there exist Banach spaces $C(X)$ over a compact scattered space X such that $C(X)$ is not weakly K-analytic (even not weakly K-countably determined under Continuum Hypothesis) but the weak dual of $C(X)$ has countable tightness. This provides also an example showing that (gDF)-spaces need not be in class $\mathfrak{G}$.

Classifications of Borel sets and functions on an arbitrary topological space

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For Borel functions on a perfect normal space and, respectively, on a perfect topological space there are two famous convergence Baire classifications: the first one is due to H.Lebesgue and F.Hausdorff and the second one is due to S.Banach.

However, for an arbitrary topological space the both classifications are not valid. In this paper the Baire convergence classifications of Borel functions on an arbitrary space are given. One convergence classification starts with some family of measurable functions and the other starts with some family of *uniform* functions.

These classifications of Borel functions use two classifications of Borel sets; one of them generalizes the Young-Hausdorff classification for a perfect space, the other is new.

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A theorem in memory of Susanne Dierolf

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An important result of Prof. S. Dierolf says that every barrelled locally convex topological space E with $E' \neq E^*$ has a dense infinite-codimensional subspace [4.6.6 in Prez Carreras/Bonet's book]. A recent preprint [Saxon, *Mackey hyperplanes and enlargements for Tweddle's spaces*] properly relaxes the hypothesis: If the dense hyperplanes of a primitive space E with $E' \neq E^*$ are Mackey, then E has a dense infinite-codimensional subspace. This talk sketches the proof as it borrows from and adds to ideas of Levin/Saxon/Tweddle/Valdivia.

Category-like (barrelled/Baire–like) properties of $C_c(X)$ and bounding/compact properties in X

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For a Tychonoff space X, let $C_c(X)$ be the linear space of continuous realvalued functions on X given the compact-open topology. Moving beyond the Nachbin/Shirota characterization of barrelled $C_c(X)$, in terms of X, Lehner [L] supplied characterizations, in terms of X, for Saxon's Bairelike and unordered Bairelike properties on $C_c(X)$ [S, TS]. We report on an example, in the positive direction, to question 6.1 of [TR] which uses Kunen's weak P-points in the Stone-Cech growth $\omega^* = \beta\omega \setminus \omega$ of the non-negative integers: Specifically, there is an unordered Bairelike $C_c(X)$ with X realcompact, but not strongly Hewitt, as defined by Kakol and Sliwa [KS]. Further questions relate to the use of $C_c(X)$ to characterize X with countably compact (pseudocompact) growth $X^* = \beta X \setminus X$.

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The Controlled Separable Projection Property for Banach Spaces

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Let X be a *nonseparable* Banach space. The space X possesses the Controlled Separable Projection Property (CSPP for short) if for every two sequences (x_n) and (x_n^*) in X and X^*, respectively, there is a continuous projection P on X such that

(i) $P(X)$ is separable, and
(ii) (x_n) and (x_n^*) are contained in $P(X)$ and $P^*(X^*)$, respectively.

Every WCD-(hence, every WCG-)space has the CSPP. The notion of CSPP was introduced in 2002 by the present author [3] to the study of the structure of quotient Banach spaces. In 2004 Banakh, Plichko and Zagorodnyuk [1] studied the zero-set of a quadratic homogeneous polynomial on $X \in CSPP$. Recently, Ferrer [2] obtained a characterization of $C(K)$-spaces which enjoy the CSPP, and showed that if Y is a closed subspace of $X \in CSPP$ such that $(X/Y)^*$ is weak*-separable, then $Y \in CSPP$.

I shall present a few results on CSPP-spaces in the context of the Separable Quotient Problem, e.g., the one below completes the above-cited result by Ferrer.

Theorem. *Let Y be a closed subspace of $X \in CSPP$. If the space $(X/Y)^*$ is weak*-separable, then there is a separable and complemented* *subspace F of X such that X/Y is a quotient of F; consequently, X/Y is separable.*

Corollary. *Let $X \in CSPP$, and let (x_n^*) be a linearly independent sequence in X^*. Then the quotient space $X/\bigcap \mathrm{ker} x_n^*$ is separable and of infinite dimension.*

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Topology and Dynamical Systems

Limits of Inverse Limits - Examples

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We present examples of limits of sequences of inverse limits with closed intervals and usc set-valued bonding functions with respect to the Hausdorff metric.

Shadowing, internal chain transitivity and omega-limit sets

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We discuss the role played by the pseudo-orbit tracing property (shadowing) in determining when internally chain transitive sets are omega-limit sets.

Code and order equivalence in polygonal billiards

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We compare two equivalence relations on polygonal billiards. We show when code/order equivalent billiards have the same angles, resp. are similar, resp. affinely similar.

On Ingram's Conjecture

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Ingram's conjecture states that the inverse limit spaces of two tent-maps with different slopes have non-homeomorphic inverse limit spaces. In this talk, I will present some new developments emerging from joint work with Barge and Štimac.

Limits of inverse limits

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We present the following problem: if a sequence of graphs of upper semicontinuous set-valued functions f_n converges to the graph of a function f, is it true that the sequence of corresponding inverse limits obtained from f_n converges to the inverse limit obtained from f?

Uncountable omega-limit sets with isolated points

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The omega limit set W of (the critical point of) a tent map on $[0, 1]$ is a compact subset of the interval. In the countable case, we have an exact topological description of W. In the uncountable case, W may be a Cantor set, or the whole of the interval, or may be of intermediate type, containing a Cantor set but also some other structure, including isolated points. We discuss the structure in this latter case, for example in this case the Cantor set may or may not be minimal.

Almost totally disconnected minimal systems

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In this talk I start with a brief survey of old and recent results on topological structure of minimal sets in dynamical systems. Then I present new results from a joint paper by Balibrea, Downarowicz, Hric, Snoha and pitalsk, of the same title as the talk. We construct a new rich class of minimal systems - almost totally disconnected minimal systems. A topological space is said to be almost totally disconnected if the set of its degenerate components is dense. We prove that an almost totally disconnected compact metric space admits a minimal map if and only if either it is a finite set or it has no isolated point. As a consequence we obtain a topological characterization on minimal sets on dendrites and local dendrites. We also prove that any infinite compact almost totally disconnected space with no isolated point admits a minimal map with arbitrary entropy.

Dynamics of Induced Maps to Symmetric Products

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Given a metric continuum X, the nth-symmetric product of X is defined as $F_n(X) = \{A : A$ is a nonempty subset of X with at most n elements$\}$, $F_n(X)$ is endowed with the Vietoris topology. Given a continuous function f from X to X, the induced function f_n is the map from $F_n(X)$ into $F_n(X)$ given by $f_n(A) = f(A)$ (the image of A under f). In this talk we discuss how some dynamic properties of the map f are translated to dynamic properties of the map f_n and vice versa.

Inverse limits and the problem of backward dynamics in economics

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In several standard economics models, the problem of "backward dynamics" arises, i.e., from the model a map arises which is well defined going backward in time, but it is not well defined going forward in time. Of course, economists would like to predict the future, and need techniques that can handle this situation. We use inverse limits to overcome this problem, and have obtained both quantitative and quantitative results that apply to dynamic equilibrium models.

Entropy, horseshoes and homoclinic trajectories on trees, graphs and dendrites

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It is known that the positiveness of topological entropy, the existence of a horseshoe and the existence of a homoclinic trajectory are mutually equivalent, for interval maps. The aim of the talk is to describe the relations between the properties for continuous maps of trees, graphs and dendrites. We consider three different definitions of a horseshoe and two different definitions of a homoclinic trajectory. For example, positive topological entropy and the existence of a homoclinic trajectory are independent and neither of them implies the existence of any horseshoe in the case of dendrites. Unfortunately, still there is an open problem, and we formulate it at the end of the talk.

Blockers in hyperspaces

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A closed subset B of a metric continuum X *blocks* a nonempty closed set $A \subset X$ if, for each continuous path p from $[0,1]$ into the hyperspace 2^X (of closed nonempty subsets of X with the Hausdorff metric), there is $t < 1$ such that $p(t) \cap B \neq \emptyset$. Blockers can be viewed as a generalization of closed separators. We prove that if X is a nondegenerate locally connected continuum such that no finite subset separates X, then the family $\mathbf{B}$ of all sets that block each subcontinuum of X is a capset in the Hilbert cube 2^X. In particular, $\mathbf{B}$ is homeomorphic to the pseudoboundary of of the Hilbert cube $[0,1]^\infty$.

Bounded orbits and preserving measure

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In 1996, G. Kuperberg proved that every boundaryless 3-manifold M admits a measure preserving C^∞ non-singular flow. An orbit is *bounded* if its closure is compact. The methods can be modified to obtain a measure preserving C^∞ non-singular flow with all orbits bounded on any boundaryless 3-manifold.

Let $\mathcal{U}$ be an open cover of M. A flow has orbits bounded by $\mathcal{U}$ if the closure of every of its orbits is contained in some element of $\mathcal{U}$. It is not known if every boundaryless 3-manifold M with a given open cover $\mathcal{U}$ admits a nonsingular measure preserving flow with orbits bounded by $\mathcal{U}$.

In particular, the question of G. Kuperberg, whether there exists a measure preserving non-singular flow on $\mathbb{R}^3$ with all orbits of diameter smaller than one, remains unanswered.

Laminations of the Unit Disk and Julia Sets

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A complex polynomial P of degree ≥ 2 always has a nonempty, perfect, nowhere dense fully invariant subset $J(P)$ of the complex plane called its *Julia set.* On this set the map P is *chaotic.* By way of a classical theorem on change of variables around a fixed critical point, the Böttkher Uniformization Theorem, a connection can be made between the topological and dynamical structure of (connected) Julia sets and invariant laminations of the unit disk, potentially a friendlier object of study. For example, if we parameterize the unit circle by $[0, 1)$, the map on the unit circle which corresponds to a cubic polynomial is $t \mapsto 3t \pmod 1$, a (deceptively) very simple map, until one considers what its invariant subsets on the circle are. In this talk, we define invariant laminations of the unit disk and their relation, in particular, to quadratic and cubic Julia sets. We use this as a jumping-off point for open questions about Julia sets and invariant laminations, and recent progress made on some of them.

Dynamics of commuting maps of chainable continua

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A chainable continuum, X, and homeomorphisms, $p, q : X \longrightarrow X$, are constructed with the following properties:

1. $p \circ q = q \circ p$
2. p, q have simple dynamics
3. $p \circ q$ is a positively continuum-wise fully expansive homeomorphism that has positive entropy and is chaotic in the sense of both Devaney and Li-Yorke.

Other related questions and properties are also explored.

On homogeneous Suslinian continua

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A (Hausdorff) continuum is Suslinian if it does not contain uncountably many pairwise disjoint non-degenerate subcontinua. Existence of non-metrizable Suslinian continua is equivalent to the negation of the Suslin Hypothesis. However, no set-theoretic conditions are needed to prove that (a) a homogeneous and non-degenerate Suslinian continuum is a simple closed curve, and (b) each separable and homogeneous Suslinian continuum is metrizable.

Shadowing, entropy and a homeomorphism of the pseudoarc

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In this talk we present a method of construction of continuous map $f\colon [0,1] \to [0,1]$, such that f is topologically mixing, has the shadowing property and the inverse limit of copies of $[0,1]$ with f as the bounding map is the pseudoarc. This map indeuces a homeomorphism of the pseudoarc with the shadowing property and positive topological entropy. We therefore answer, in the affirmative, a question posed by Chen and Li in 1993 whether such a homeomorphism exists.

Combinatorial Classification of Cubic Polynomials with a fixed Siegel Disk

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In this poster we will consider the dynamics of polynomial maps of the form $z \mapsto \lambda z + a_1 z^2 + a_2 z^3$ where $\lambda = e^{2\pi i\theta}$ and $0 < \theta < 1$ is an irrational number of Brjuno type. In this case, the map is linearizable in a neighborhood of the fixed point at 0, and the maximal such neighborhood is called a Siegel Disk. One of the critical points of the map always accumulates on the boundary of the Siegel Disk. Next, we adopt the parameterization of Zakeri by first marking the critical points such that the first critical point under the marking is always 1 and then taking affine conjugacy classes which respect the marking. A cubic with the above form which has its second critical point at c has the form $P_c : z \mapsto \lambda z(1 - \frac{1}{2}(1 + \frac{1}{c})z + \frac{1}{3c}z^2)$. Zakeri defines the cubic connectedness locus $\mathcal{M}(\theta)$ to be points $c \in \mathbb{C}$ such that the orbits of 1 and c under P_c are bounded. The components of the interior of $\mathcal{M}(\theta)$ are classified by how the critical point not necessarily associated with the Siegel disk behaves, also called the "free" critical point. Of particular interest to us are the so-called *capture* components, in which the free critical point eventually maps into the Siegel Disk. Let $\mathcal{C}$ be the union of the closures of all capture components. We define the Principal Capture Locus to be the largest component of $\mathcal{C}$. We first define a natural numbering of the capture components that make up $\mathcal{C}$ and then develop a combinatorial classification of the dynamics of maps $P_c, c \in \mathcal{C}$ using laminations, a simply connected model for polynomial dynamics.

On Ingram's Conjecture

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In recent years there has been intensive research of topological properties of inverse limit spaces of tent maps with classification of these spaces as ultimate goal. The Ingram conjecture claims that two inverse limits of tent maps with different slopes are not homeomorphic. I will discuss recent progress on Ingram's conjecture emerging from joint work with Barge and Bruin.

Transverse Foliations to nonsingular Morse-Smale flows and Bott-integrable Hamiltonian systems

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We apply results of Goodman, Yano and Wada to determine which nonsingular Morse-Smale flows on the 3-sphere have transverse foliations. We then observe that there is a connection to flows arising from certain Hamiltonian systems and from certain contact structures. Recent work by Cordero et al might allow us to extend these results to $S^2 \times S^1$.

Remarks on Countable Rank Maps

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Let $f : X \longrightarrow Y$ be a function and let $\mathfrak{m}$ be an infinite cardinal. Then we say that the rank of f is $\leq \mathfrak{m}$ if $|\{y \in Y : |f^{-1}(y)| > 1\}| \leq \mathfrak{m}$. If $\mathfrak{m} = \aleph_0$, then f is of countable rank. In this talk, some results concerning projective classes of countable rank maps will be presented.

Maximal Dendrtes Embedded in Locally Connected Continua

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We show that there exists a class of dendrtes C, such that for any n greater than 2, and any locally connected continuum X of dimension n, all dendrites of class C can be embedded in X, and that there exists a locally connected continua Y_n of dimension n, such that any dendrite that is not in C, can not be embedded in Y_n. This result gives us a characterization of dynamical properties of locally connected continua.

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