Vypočítejte $\int_0^\infty f(x)\,dx$, kde $f(x)$ je funkce, kterou jste v 1. úloze rozkládali na parciální zlomky (rozklad znovu neprovádějte). Použijte již rozložený tvar.

1. $f(x)=\dfrac{3x^3+x^2-4x+16}{x^5+5x^4+9x^3+13x^2+14x+6}=\dfrac{1}{x+1}+\dfrac{3}{(x+1)^2}-\dfrac{1}{x+3}-\dfrac{2}{x^2+2}$,

$$\int_0^\infty\left(\frac{1}{x+1}+\frac{3}{(x+1)^2}-\frac{1}{x+3}-\frac{2}{x^2+2}\right)dx=\left[\ln(x+1)-\frac{3}{x+1}-\ln(x+3)-\sqrt{2}\,\mathrm{arctg}\,\frac{x}{\sqrt{2}}\right]_0^\infty=$$

$$=\lim_{x\to\infty}\left(\ln\frac{x+1}{x+3}-\frac{3}{x+1}-\sqrt{2}\,\mathrm{arctg}\,\frac{x}{\sqrt{2}}\right)-\ln\frac{1}{3}+3=\underline{\underline{3+\ln 3-\frac{\pi\sqrt{2}}{2}}}$$

2. $f(x)=\dfrac{-x^3-11x^2+8x+30}{x^5+7x^4+17x^3+23x^2+30x+18}=\dfrac{1}{x+1}+\dfrac{1}{x+3}+\dfrac{3}{(x+3)^2}-\dfrac{2x}{x^2+2}$

$$\int_0^\infty\left(\frac{1}{x+1}+\frac{1}{x+3}+\frac{3}{(x+3)^2}-\frac{2x}{x^2+2}\right)dx=\left[\ln(x+1)+\ln(x+3)-\frac{3}{x+3}-\ln(x^2+2)\right]_0^\infty=$$

$$=\lim_{x\to\infty}\left(\ln\frac{(x+1)(x+3)}{x^2+2}-\frac{3}{x+3}\right)-\ln\frac{3}{2}+1=\underline{\underline{1-\ln\frac{3}{2}}}$$

3. $f(x)=\dfrac{2x^3+7x^2-7}{x^5+5x^4+9x^3+9x^2+8x+4}=\dfrac{2x}{x^2+1}-\dfrac{1}{x+1}-\dfrac{1}{x+2}-\dfrac{1}{(x+2)^2}$

$$\int_0^\infty\left(\frac{2x}{x^2+1}-\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{(x+2)^2}\right)dx=\left[\ln(x^2+1)-\ln(x+1)-\ln(x+2)+\frac{1}{x+2}\right]_0^\infty=$$

$$=\lim_{x\to\infty}\left(\ln\frac{x^2+1}{(x+1)(x+2)}+\frac{1}{x+2}\right)-\ln\frac{1}{2}-\frac{1}{2}=\underline{\underline{\ln 2-\frac{1}{2}}}$$

4. $f(x)=\dfrac{-x^3-6x^2+x+8}{x^5+5x^4+9x^3+9x^2+8x+4}=\dfrac{1}{x+1}+\dfrac{1}{x+2}+\dfrac{2}{(x+2)^2}-\dfrac{2x}{x^2+1}$

$$\int_0^\infty\left(\frac{1}{x+1}+\frac{1}{x+2}+\frac{2}{(x+2)^2}-\frac{2x}{x^2+1}\right)dx=\left[\ln(x+1)+\ln(x+2)-\frac{2}{x+2}-\ln(x^2+1)\right]_0^\infty=$$

$$=\lim_{x\to\infty}\left(\ln\frac{(x+1)(x+2)}{x^2+1}-\frac{2}{x+2}\right)-\ln 2+1=\underline{\underline{1-\ln 2}}$$

5. $f(x)=\dfrac{2x^3+9x^2+16x+7}{x^5+5x^4+9x^3+9x^2+8x+4}=-\dfrac{1}{x+1}+\dfrac{1}{x+2}+\dfrac{1}{(x+2)^2}+\dfrac{2}{x^2+1}$

$$\int_0^{\infty}\left(-\frac{1}{x+1}+\frac{1}{x+2}+\frac{1}{(x+2)^2}+\frac{2}{x^2+1}\right)dx=\left[-\ln(x+1)+\ln(x+2)-\frac{1}{x+2}+2\operatorname{arctg}x\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(\ln\frac{x+2}{x+1}-\frac{1}{x+2}+2\operatorname{arctg}x\right)-\ln 2+\frac{1}{2}=\underline{\underline{\pi-\ln 2+\frac{1}{2}}}$$

6. $$f(x)=\frac{-2x^3-14x^2-6x+14}{x^5+7x^4+16x^3+16x^2+15x+9}=\frac{1}{x+1}+\frac{1}{x+3}+\frac{2}{(x+3)^2}-\frac{2x}{x^2+1},$$

$$\int_0^{\infty}\left(\frac{1}{x+1}+\frac{1}{x+3}+\frac{2}{(x+3)^2}-\frac{2x}{x^2+1}\right)dx=\left[\ln(x+1)+\ln(x+3)-\frac{2}{x+3}-\ln(x^2+1)\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(\ln\frac{(x+1)(x+3)}{x^2+1}-\frac{2}{x+3}\right)-\ln 3+\frac{2}{3}=\underline{\underline{\frac{2}{3}-\ln 3}}$$

7. $$f(x)=\frac{9x^3+19x^2+x-5}{x^5+5x^4+8x^3+8x^2+7x+3}=\frac{1}{(x+1)^2}-\frac{2}{x+1}-\frac{2}{x+3}+\frac{4x}{x^2+1}$$

$$\int_0^{\infty}\left(\frac{1}{(x+1)^2}-\frac{2}{x+1}-\frac{2}{x+3}+\frac{4x}{x^2+1}\right)dx=\left[-\frac{1}{x+1}-2\ln(x+1)-2\ln(x+3)+2\ln(x^2+1)\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(-\frac{1}{x+1}+2\ln\frac{x^2+1}{(x+1)(x+3)}\right)+1-2\ln\frac{1}{3}=\underline{\underline{1+2\ln 3}}$$

8. $$f(x)=\frac{x^3+19x^2+25x+11}{x^5+5x^4+8x^3+8x^2+7x+3}=\frac{1}{(x+1)^2}-\frac{2}{x+1}+\frac{2}{x+3}+\frac{4}{x^2+1}$$

$$\int_0^{\infty}\left(\frac{1}{(x+1)^2}-\frac{2}{x+1}+\frac{2}{x+3}+\frac{4}{x^2+1}\right)dx=\left[-\frac{1}{x+1}-2\ln(x+1)+2\ln(x+3)+4\operatorname{arctg}x\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(-\frac{1}{x+1}+2\ln\frac{x+3}{x+1}+4\operatorname{arctg}x\right)+1-2\ln 3=\underline{\underline{2\pi+1-2\ln 3}}$$

9. $$f(x)=\frac{9x^3+33x^2+17x-23}{x^5+7x^4+16x^3+16x^2+15x+9}=\frac{1}{(x+3)^2}-\frac{2}{x+3}-\frac{2}{x+1}+\frac{4x}{x^2+1}$$

$$\int_0^{\infty}\left(\frac{1}{(x+3)^2}-\frac{2}{x+3}-\frac{2}{x+1}+\frac{4x}{x^2+1}\right)dx=\left[-\frac{1}{x+3}-2\ln(x+3)-2\ln(x+1)+2\ln(x^2+1)\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(-\frac{1}{x+3}+2\ln\frac{x^2+1}{(x+3)(x+1)}\right)+\frac{1}{3}-2\ln\frac{1}{3}=\underline{\underline{\frac{1}{3}+2\ln 3}}$$

10. $f(x)=\dfrac{3x^3+17x^2+31x+13}{x^5+5x^4+9x^3+9x^2+8x+4}=\dfrac{1}{(x+2)^2}+\dfrac{2}{x+2}-\dfrac{2}{x+1}+\dfrac{4}{x^2+1}$

$$\int_0^{\infty}\left(\frac{1}{(x+2)^2}+\frac{2}{x+2}-\frac{2}{x+1}+\frac{4}{x^2+1}\right)dx=\left[-\frac{1}{x+2}+2\ln(x+2)-2\ln(x+1)+4\operatorname{arctg}x\right]_0^{\infty}=$$

$$=\lim_{x\to\infty}\left(-\frac{1}{x+2}+2\ln\frac{x+2}{x+1}+4\operatorname{arctg}x\right)+\frac{1}{2}-2\ln 2=\underline{\underline{2\pi+\frac{1}{2}-2\ln 2}}$$