Limity postupnosti

1.
$$\lim_{n\to\infty} \frac{3-5n^2-2n^4}{3-2n^5} = \lim_{n\to\infty} \frac{\frac{3}{n^5}-\frac{5}{n^3}-\frac{2}{n}}{\frac{3}{n^5}-2} = \frac{0-0-0}{0-2} = 0.$$

2.
$$\lim_{n\to\infty} \frac{2n^2+3n-1}{3n^2-2n+1} = \lim_{n\to\infty} \frac{2+\frac{3}{n}-\frac{1}{n^2}}{3-\frac{2}{n}+\frac{1}{n^2}} = \frac{2+0-0}{3-0+0} = \frac{2}{3}.$$

3.
$$\lim_{n\to\infty} \frac{2n^2+1}{3n^2+n} = \lim_{n\to\infty} \frac{n^2\left(2+\frac{1}{n^2}\right)}{n^2\left(3+\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{2+\frac{1}{n^2}}{3+\frac{1}{n}} = \frac{2}{3}$$

4.
$$\lim_{n\to\infty} \left(\frac{n}{2}+\frac{1}{3}\right) = \infty$$

5.
$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^r = \lim_{n\to\infty} \left(\frac{1}{1+\frac{1}{n}}\right)^r = \left(\frac{1}{1+0}\right)^r = 1.$$

6.
$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n+5} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \cdot \left(1+\frac{1}{n}\right)^5 = e.1 = e$$

7.
$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{3n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^3 = e^3$$

8.
$$\lim_{n\to\infty} \left(1+\frac{1}{5n}\right)^n = \lim_{n\to\infty} \left[\left(1+\frac{1}{5n}\right)^{5n}\right]^{\frac{1}{5}} = e^{\frac{1}{5}}$$

9.
$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-n} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e}.$$

10.
$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty} \left(\frac{n}{n-1}\right)^{-n} = \frac{1}{\lim_{n\to\infty} \left(1+\frac{1}{n-1}\right)^{n-1} \cdot \left(1+\frac{1}{n-1}\right)} = \frac{1}{e}.$$

11.
$$\lim_{n\to\infty} \left(1+\frac{1}{2n+3}\right)^{7n+6} = \lim_{n\to\infty} \left(1+\frac{1}{2n+3}\right)^{\frac{7}{2}(2n+3)-\frac{9}{2}} = e^{\frac{7}{2}}$$

12.
$$\lim_{n\to\infty} \left(\frac{2n}{n-1}\right)^{2n} = \lim_{n\to\infty} 2^{2n} \cdot \left(\frac{n}{n-1}\right)^{2n} = \lim_{n\to\infty} 4^n \cdot \left[\left(1+\frac{1}{n-1}\right)^{n-1} \cdot \left(1+\frac{1}{n-1}\right)\right]^2 = \infty$$

13.

$$\lim_{n\to\infty} \frac{2n+\sin n}{3n-1} = \lim_{n\to\infty} \frac{n\left(2+\frac{\sin n}{n}\right)}{n\left(3-\frac{1}{n}\right)} = \frac{2}{3}$$

14.

$$\lim_{n\to\infty} \left(\sqrt{n+2}-\sqrt{n}\right) = \lim_{n\to\infty} \left(\sqrt{n+2}-\sqrt{n}\right) \cdot \frac{\sqrt{n+2}+\sqrt{n}}{\sqrt{n+2}+\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{n+2-n}{\sqrt{n+2}+\sqrt{n}} = \lim_{n\to\infty} \frac{2}{\sqrt{n+2}+\sqrt{n}} = 0$$

15.

$$\lim_{n\to\infty} \left(\frac{1}{n} \cdot \cos \frac{n^2+1}{2n-1}\right) = 0$$

Zrejme

$$-1 \leq \cos \frac{n^2+1}{2n-1} \leq 1 \Rightarrow -\frac{1}{n} \leq \frac{1}{n} \cdot \cos \frac{n^2+1}{2n-1} \leq \frac{1}{n}$$

$\{-\frac{1}{n}\}_{n=1}^{\infty}, \{\frac{1}{n}\}_{n=1}^{\infty}$ konvergujú k 0, z vety o dvoch policajtoch teda vyplýva, že aj pôvodná limita je 0.